Challenge Problem: To make Pascal's Triangle, we used the identity

$$C(n,k) = C(n-1,k-1) + C(n-1,k)$$

Justify that

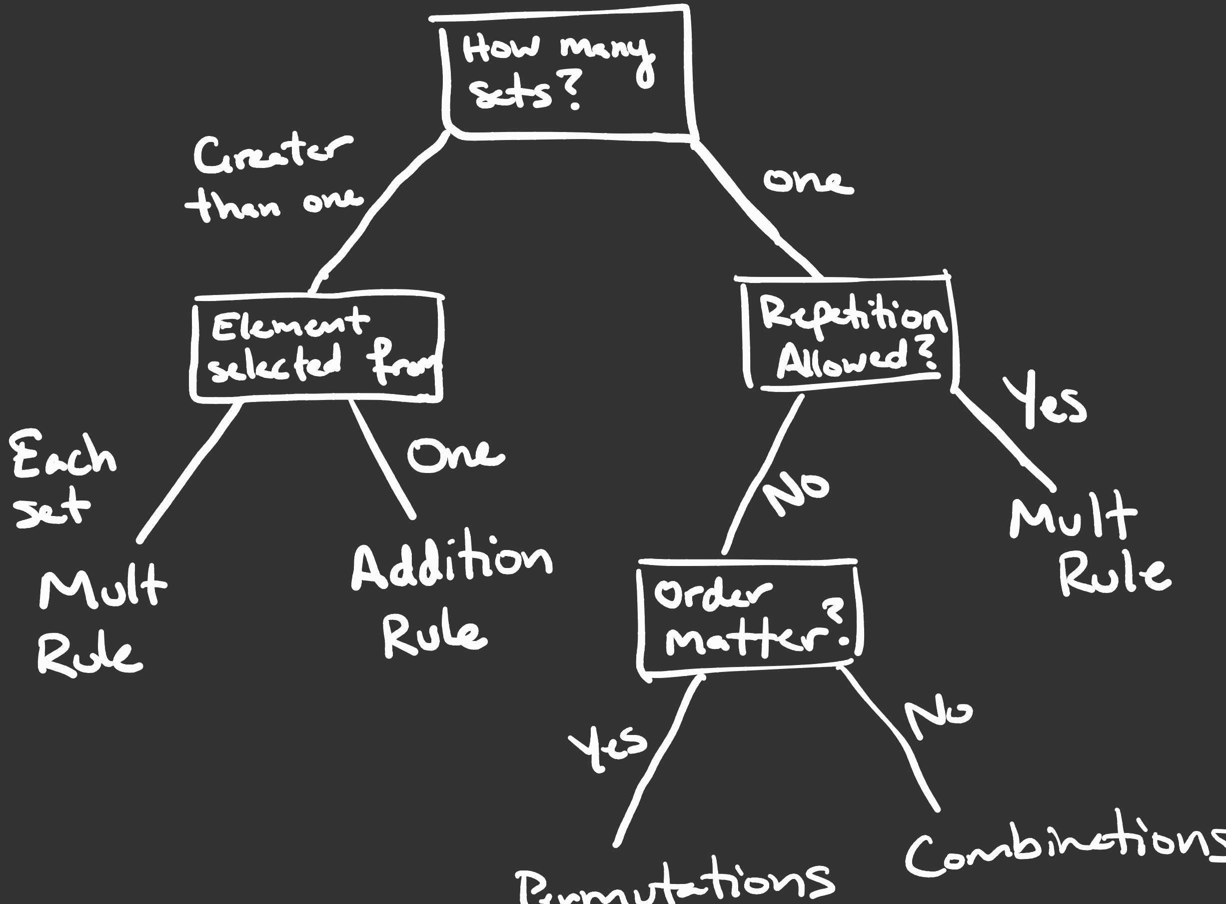
$$C(10,4) = C(9,3) + C(9,4)$$

using counting argument.

$A = \{a, b, c, \ldots, j\}$

$C(10,4) =$ # of ways to choose 4 elements from A $= \begin{pmatrix}\text{\# of 4-element} \\ \text{subsets w/ } j\end{pmatrix} + \begin{pmatrix}\text{\# subsets} \\ \text{w/o } j\end{pmatrix}$

$$= C(9,3) + C(9,4)$$


How many sets?
Greater than one
one
Element selected from
Each set
One
Mult Rule
Addition Rule
Repetition Allowed?
Yes
No
Mult Rule
Order Matter?
Yes
No
Permutations
Combinations

Example: Social Security Numbers. 9 numbers, rep allowed

a) How many w/ no repeated numbers in them?

$$P(10,9)$$

$$\rightarrow \underline{10} \times \underline{9} \times \underline{8} \times \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}$$

b) How many SSN's w/ at least one repeated number?

$$\left(\#\begin{matrix}\text{SSNS w/}\\ \geq 1 \text{ repeat}\end{matrix}\right) = \left(\begin{matrix}\#\text{total}\\ \text{SSN's}\end{matrix}\right) - \left(\begin{matrix}\#\text{SSNS}\\ \text{w/ no repeats}\end{matrix}\right)$$

$$\boxed{= 10^9 - P(10,9)}$$

c) How many SSNs w/ exactly repeated digit (showing up twice)?

(ex: 213987156)

Choose spaces w/ repeated digits → Digit to put those spaces → Fill in rest of numbers

$$\boxed{C(9,2) \times 10 \times P(9,7)}$$

Choose an 8-digit number w/ no rep → Pick a digit to repeat → Insert digit into number

$$P(10,8) \times 8 \times 10$$

$123 \rightarrow 1231$

$231 \rightarrow 1231$

Overcounted

Examples: Billy has 43 baseball cards, Scottie has 36.
How many ways can they make a 2-2 trade

$$C(43,2) \times C(36,2)$$

Sets: >1 (Billy's cards, Scottie's cards)
Selected from: Each set.

Mult Rule

Example: Sequences of 4 letters (order matters). How many sequences contain ≤ 2 A's. Repetition is allowed

26^4 = Total number of sequences

$$\begin{pmatrix}\# \text{ sequences} \\ \text{w/ } \leq 2 \text{ A's}\end{pmatrix} = \begin{pmatrix}\# \text{ total} \\ \text{sequences}\end{pmatrix} - \begin{pmatrix}\# \text{ sequences} \\ \text{w/ } 3 \text{ A's}\end{pmatrix} - \begin{pmatrix}\# \text{ sequences} \\ \text{w/ } 4 \text{ A's}\end{pmatrix}$$

$$= 26^4 - (25 \times 1 \times 1 \times 1 \times 4) - (1)$$

another option

$$C(4,3) \times 25$$

Choose spaces for A's

fill in last letter