Binomial Theorem:

$$(x+y)^n = C(n,0)\,x^n + C(n,1)\,x^{n-1}y + C(n,2)\,x^{n-2}y^2 + \cdots$$

$$\cdots + C(n,n-1)\,xy^{n-1} + C(n,n)\,y^n$$

$$(x+y)^4 = \underline{C(4,0)}\,x^4 + \underline{C(4,1)}\,x^3y + \underline{C(4,2)}\,x^2y^2 + \underline{C(4,3)}\,xy^3 + \underline{C(4,4)}\,y^4$$

Question: How many subsets does a set of size 4 have?

0-elements : $1 = C(4,0)$
1-elements : $4 = C(4,1)$
2-elements : $6 = C(4,2)$
3-elements : $4 = C(4,3)$
4-elements : $1 = C(4,4)$

$$\# \text{ of subsets} = \underline{C(4,0)} + \underline{C(4,1)} + \underline{C(4,2)} + \underline{C(4,3)} + \underline{C(4,4)}$$

Plug in

$$\xrightarrow[x=1,\, y=1]{} (1+1)^n = \underbrace{C(4,0) + C(4,1) + C(4,2) + C(4,3) + C(4,4)}$$

$$2^n = \text{\# of subsets of a set of size } 4$$

In general, $\left(\text{\# subsets of set of size } n\right) = 2^n$

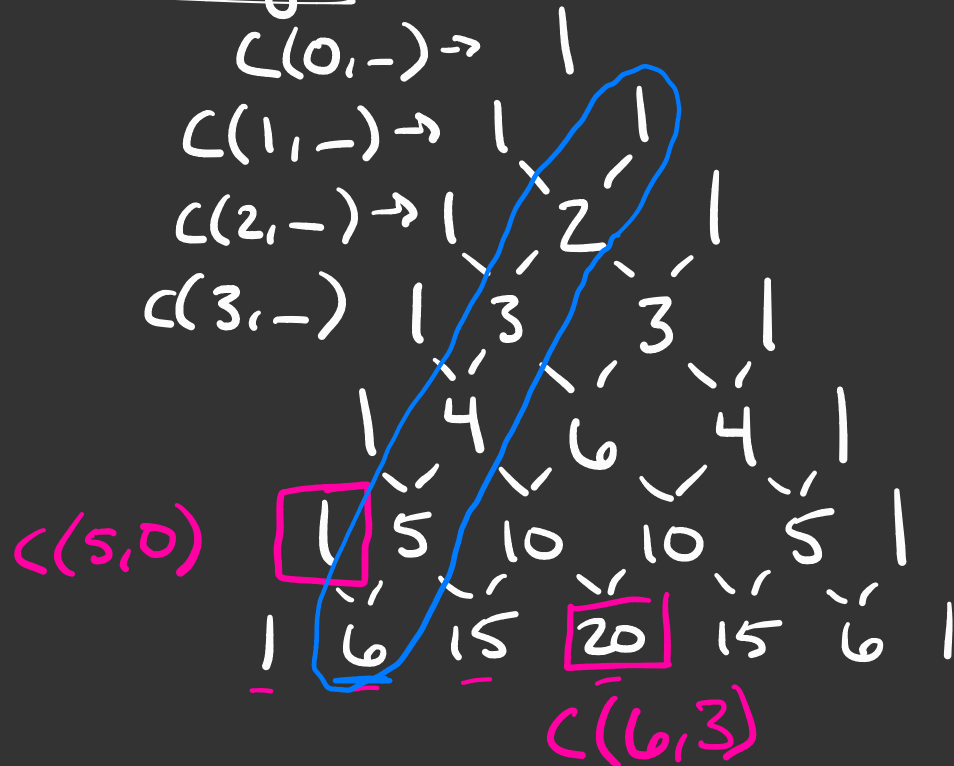
Example: How many subsets of a set of size 8 have ≥ 3 elements?

1 way: $C(8,3) + C(8,4) + \cdots + C(8,7) + C(8,8)$

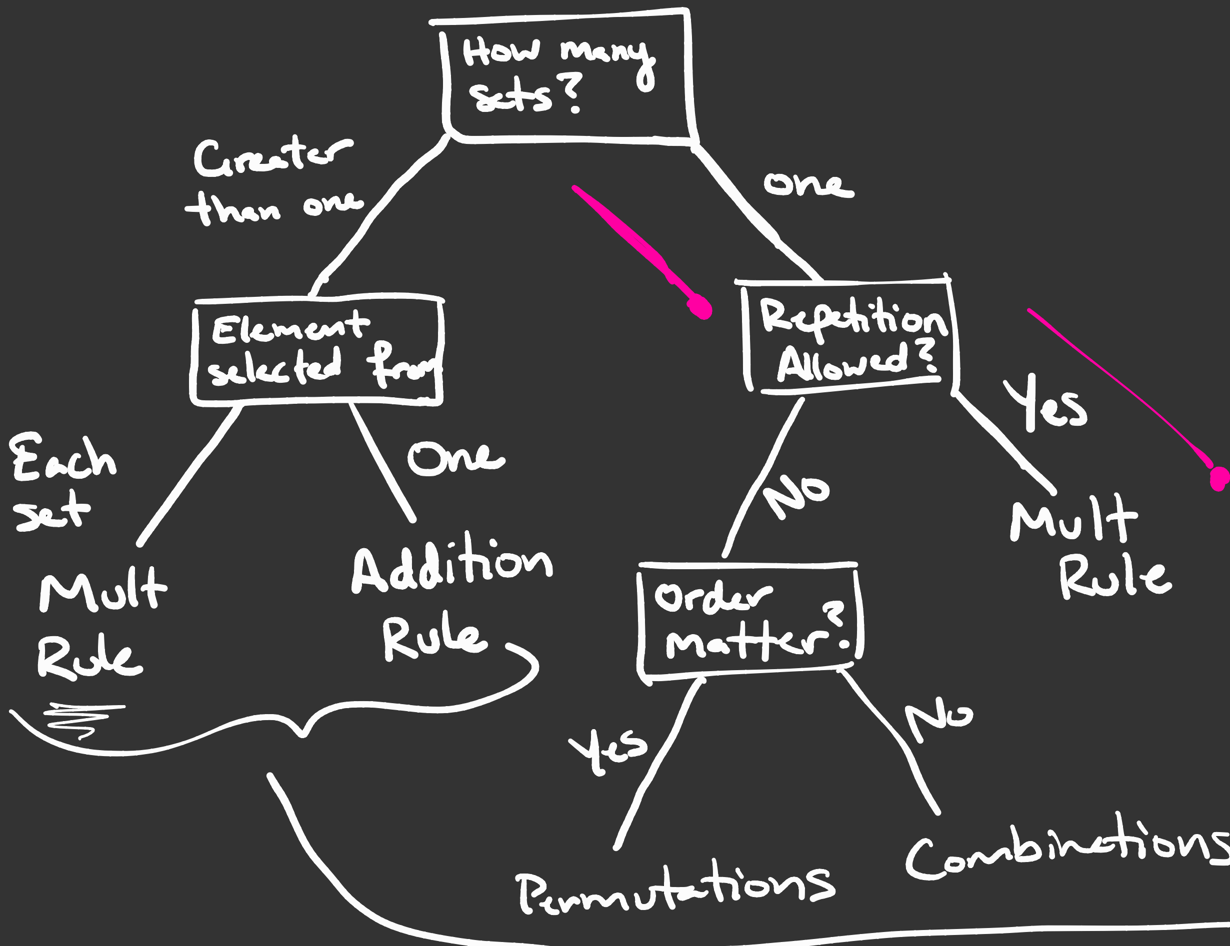
2 way: $2^8 - C(8,0) - C(8,1) - C(8,2)$

$= 256 - 1 - 8 - 28 = \underline{\underline{219}}$

Pascal's Triangle



Methods of counting:
- Multipliation Rule
- Addition Rule
- Permutation
- Combination



Note: Might have to use chart more than once in problem

Example: PIN numbers. 4 digit number (digits 0-9). Repetition allowed.

a) How many PINs are possible

sets: 1 $\{0,1,2,\dots,9\}$

rep allowed: Yes

$$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = \boxed{10{,}000}$$

b) How about if rep not allowed?

sets: 1 $\{0,\dots,9\}$

rep allowed: No

order matters: Yes

$$\boxed{P(10,4)} = 10 \cdot 9 \cdot 8 \cdot 7 = 5{,}040$$

c) How many PINS w/ at least one digit repeated?

$$\#\text{PINS w/ at one repeat} = \left(\#\text{ total PINS}\right) - \left(\#\text{ of PIN w/ no rep}\right) = 10{,}000 - 5{,}040 = \boxed{4{,}960}$$

Example: High School competition. 40 seniors, 38 juniors, 45 sophs, 37 freshman. Each class sends 4 people to complete. How many ways to pick competitors?

Sets: > 1 (freshman, sophs, juniors, seniors)

selected from: Each set

$$\left(\begin{matrix}\#\text{ ways to}\\ \text{choose}\\ \text{competitors}\end{matrix}\right) = \left(\begin{matrix}\#\text{ ways to}\\ \text{choose freshman}\end{matrix}\right) \times \left(\begin{matrix}\#\\ \text{sophs}\end{matrix}\right) \times \left(\begin{matrix}\#\\ \text{juns}\end{matrix}\right) \times \left(\begin{matrix}\#\\ \text{sens}\end{matrix}\right)$$

$$= C(37,4) \times C(45,4) \times C(38,4) \times C(40,4)$$