Challenge Problem: How many subsets does a set of size n have?

$n(n-1)(n-2)\cdots(k)$

$\left.\begin{array}{l} \text{1 element subsets : } n \\ \text{2 element subsets : } \dfrac{n(n-1)}{2} \\ \vdots \end{array}\right\}$ Add all together

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$A_1 \to A_2 \to A_3 \to \cdots \to A_n$

deciding if first element is in set

$2 \times 2 \times 2 \times \quad \times 2 = 2^n$

Motivation Example: For a Christmas Party, brought 5 different presents, but only 3 kids showed up. How many to give each kid one present?

$$5 \times 4 \times 3 = \boxed{P(5,3)} = {}_5P_3$$

"Permutation of 5 things taken 3 at a time"

Permutation: subset of distinct elements selected from a given set arranged in a specific order

Ways to recognize:

1) Elements are selected from single set
(set of presents)

2) Repetition not allowed (One present can't be given to multiple kids)

3) Order is important

Sarah - Bike
David - Skateboard
Joe - Scooter

Different

Sarah - Skateboard
David - Bike
Joe - Scooter

Examples:

i) 15 Olympians in an event. How many ways to award gold/silver/bronze medals?

Permutation! $15 \times 14 \times 13 = P(15, 3)$

ii) 22 students in class. How many ways to assign letter grade to each student (out of $\{A, B, C, D, F\}$)?

Not permutation Breaks rule 2

iii) How many 3-element subsets does a set of size 8 have?

Not permutation Breaks rule 3

Notation: $P(15,3) = \underbrace{15 \times 14 \times 13}_{3 \text{ numbers}} = 15(15-1)(15-2)$

$P(8,4) = 8 \times 7 \times 6 \times 5$

$P(n,3) = n(n-1)(n-2)$ ✓

$P(n,k) = \underbrace{n(n-1)(n-2) \cdots (n-k+1)}_{k \text{ numbers}}$

Factorial: $n! = \underline{n(n-1)(n-2)\cdots(3)(2)(1)}$

"n factorial"

Example: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Special Cases: $1! = 1$

$0! = 1$

Another way to write permutations:

$$P(8,3) = 8 \times 7 \times 6 = \frac{8 \times 7 \times 6 \times 5!}{5!} = \frac{8!}{5!}$$

General Formula

$$\boxed{P(n,k) = \frac{n!}{(n-k)!}}$$

Permutations w/ some objects alike

Example: How many way can all letters of "ABOUT" be rearranged?

$$5! = P(5,5) = 120$$

Example: How many ways can all letters of "AGREE" be rearranged

Not quite permutation (Breaks rule 3)

i) Pretend E's are different

$$\underbrace{\text{"}AGRE_1E_2\text{"}}$$

$P(5,5) = 5!$ ways to rearrange

ii) How much did we overcount? Factor of 2

ways to permute E's in a word

$G\ R\ E_1\ A\ E_2$

$G\ R\ E_2\ A\ E_1$

$\leadsto$ "AGREE" can be rearranged in $\frac{5!}{2}$ ways

Example: "DEEPEN"

i) "$DE_1E_2PE_3N$"

$P(6,6) = 6! = 720$

ii)
$E_1 \; D \; E_2 \; N \; E_3 \; P$
$E_1 \; D \; E_3 \; N \; E_2 \; P$
$E_2 \; D \; E_1 \; N \; E_3 \; P$
$E_2 \; D \; E_3 \; N \; E_1 \; P$
$E_3 \; D \; E_1 \; N \; E_2 \; P$
$E_3 \; D \; E_2 \; N \; E_1 \; P$

"BANANA" $= \dfrac{6!}{\underset{\text{A's}}{3!}\ \underset{\text{N's}}{2!}}$

Overcounted by factor of 3!

$\rightsquigarrow$ "DEEPEN" can be rearranged in $\dfrac{6!}{3!}$ ways