What if a game is <u>not</u> strictly determined?

Players don't do same <u>strat</u> every time, but rather a <u>mixture</u> of strategies.

Goal: Decide which mixture of strategies works out best

$$A \longrightarrow \begin{array}{c|cc} & c_1 & c_2 \\ \hline r_1 & a_{11} & a_{12} \\ r_2 & a_{21} & a_{22} \end{array}$$

R plays r_1 w/ prob p_1
r_2 w/ prob p_2

C plays c_1 w/ prob q_1
c_2 w/ prob q_2

Strats	r_1c_1	r_1c_2	r_2c_1	r_2c_2
Prob	p_1q_1	p_1q_2	p_2q_1	p_2q_2
Payoff	a_{11}	a_{12}	a_{21}	a_{22}

$\rightsquigarrow$ Expected Payoff $= p_1q_1a_{11} + p_1q_2a_{12} + p_2q_1a_{21} + p_2q_2a_{22}$

Can represent R and C's mixture of strategies by matrices

$$P = \begin{bmatrix} p_1 & p_2 \end{bmatrix} \qquad Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Magic: $$PAQ = \underset{1\times 2}{\begin{bmatrix} p_1 & p_2 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}$$

$$= \begin{bmatrix} p_1 q_1 a_{11} + p_1 q_2 a_{12} + p_2 q_1 a_{21} + p_2 q_2 a_{22} \end{bmatrix}$$

$=$ Expected Value $= E(P, Q)$

Example:

$$\begin{array}{cccc} & \multicolumn{2}{c}{C} & \\ & c_1 & c_2 & \text{Row Min} \\ R \begin{array}{c} r_1 \\ r_2 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} 15 & 75 \\ 45 & -30 \end{bmatrix}} & \begin{array}{c} \boxed{15} \\ -30 \end{array} \\ \text{Col Max} & \boxed{45} & 75 & \end{array}$$

$$P = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix} \qquad Q = \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}$$

$$\text{Expected Value} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix} \underbrace{\begin{bmatrix} 15 & 75 \\ 45 & -30 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}}_{2\times 1}$$

$$= \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 6+45 \\ 18+(-18) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}}_{1\times 2} \underbrace{\begin{bmatrix} 51 \\ 0 \end{bmatrix}}_{2\times 1} = \begin{bmatrix} 34 \end{bmatrix}$$

R expects to win \$34 on average

Q: How do we determine the best mixture of strategies? This is HARD

$$\begin{array}{c c} & \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{array}{c} r_1 \\ r_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

Best strategy for R: $P = [p_1 \quad p_2]$

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$p_2 = 1 - p_1$$

Best strategy for C: $Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$q_2 = 1 - q_1$$

$$\text{Expected Payoff} = PAQ = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

Example:

$$\begin{bmatrix} 15 & 75 \\ 45 & -30 \end{bmatrix}$$

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$= \frac{-30-45}{15-30-75-45} = \frac{-75}{-135} = \frac{5}{9}$$

$$p_2 = \frac{4}{9}$$

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} = \frac{-30-75}{-135} = \frac{-105}{-135}$$

$$= \frac{7}{9}$$

$$q_2 = \frac{2}{9}$$

$$\text{Expected value} = \frac{15(-30) - 75(45)}{-135}$$

$$= \frac{-3825}{-135} = 28.3$$