

Example: Coin matching

Two players, Rob and Chad

Each player has coin, and decide which face of coin to turn up (simultaneously)

Both H: Rob pays Chad 50¢

Both T: Rob pays Chad 25¢

One of each: Chad pays Rob 35¢

Can convert this to a payoff matrix

Pos : R gets paid
Neg : C gets paid

R

		C	
		H	T
R	H	-50¢	35¢
	T	35¢	-25¢

Question: Is there a strategy to these games?

Answer: Not always

Example:

$$\begin{array}{cc} & \begin{matrix} H & T \end{matrix} \\ \begin{matrix} R \\ T \end{matrix} & \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \end{array}$$

Always 50/50
chance.

Some games do have strategies.

Example:

		C	
		c_1	c_2
R	r_1	4	-9
	r_2	6	8

Strategy for R: r_2

Strategy for C: c_1 , even though their only chance of winning is with c_2 . They know that R will choose r_2

- The value of this game is 6
 - The location (2,1) is called the saddle point
 - r_2, c_1 are called solution to this game.
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Note: Payoff matrix C doesn't have to be 2×2 :

$$\begin{array}{c}
 R \\
 \begin{array}{c}
 r_1 \\
 r_2 \\
 \vdots \\
 r_m
 \end{array}
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 c_1 \quad c_2 \quad \dots \quad c_n
 \end{array} \\
 \left[\begin{array}{ccc}
 a_{11} & a_{12} & a_{1n} \\
 a_{21} & a_{22} & a_{2n} \\
 & & \\
 a_{m1} & a_{m2} & a_{mn}
 \end{array} \right]
 \end{array}$$

R has m choices
 C has n .

Assumptions:

- 1) Each player wants to win as much as possible
- 2) Neither player know what other will choose beforehand (unless there's a reason for it)
- 3) Each player assumes the other player is smart and plays rationally. (C knows R will choose r_2 every time in last example)

How to find strategy:

		C		Row Min
		c_1	c_2	
R	r_1	4	-9	-9
	r_2	6	8	6

R should choose this row

Col Max | 6 8

R's perspective: Look at each row, find minimum entry of each row.

- Select row with biggest min

C's perspective: Look at each column, find maximum entry of each column

- Select column with smallest max

If $(\text{largest row min}) = (\text{smallest col max})$, then the game is called strictly-determined.

(players should play same strategy every single time)

Example:

		C		Row
		C_1	C_2	Min
R	r_1	1	2	1
	r_2	3	4	3
	r_3	7	5	<u>5</u>
		Col Max	7 <u>5</u>	

Strictly determined: Yes

Value: 5

Saddle Point: (3, 2)

Solution: r_3, C_2