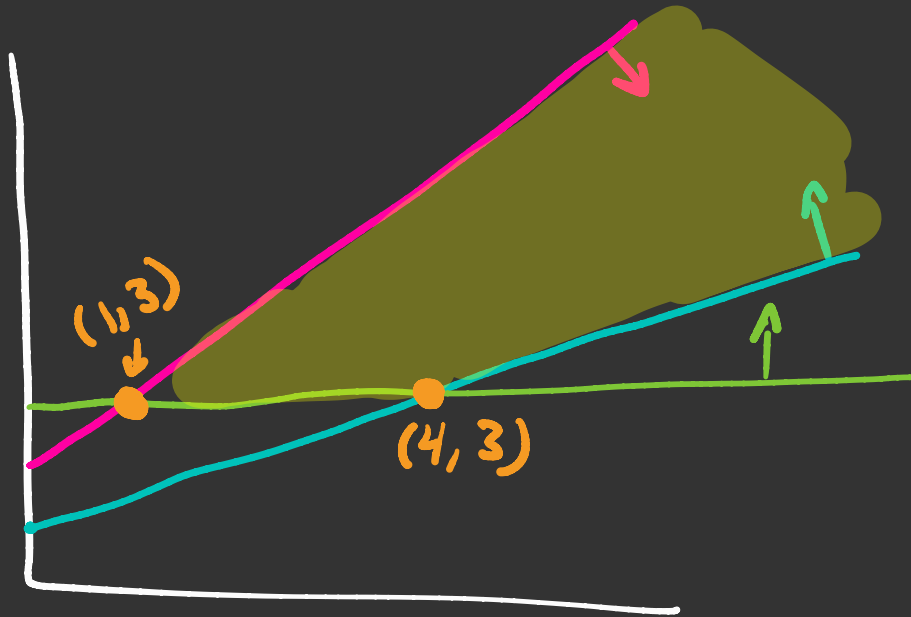


Example: Constraints :  $-x + y \leq 2$  —  
 $-x + 2y \geq 2$  —  
 $y \geq 3$

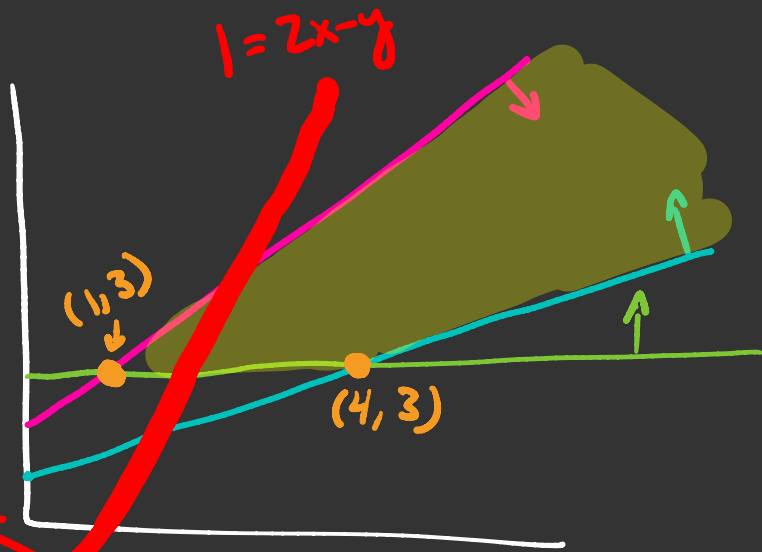


Max/min the following functions w/ these constraints

a)  $z = 2x - y$

$(1, 3) : z = -1$

$(4, 3) : z = 5$



Look at  $z = 2x - y$

Min:  $z = -1$  at  $(1, 3)$

Max: None

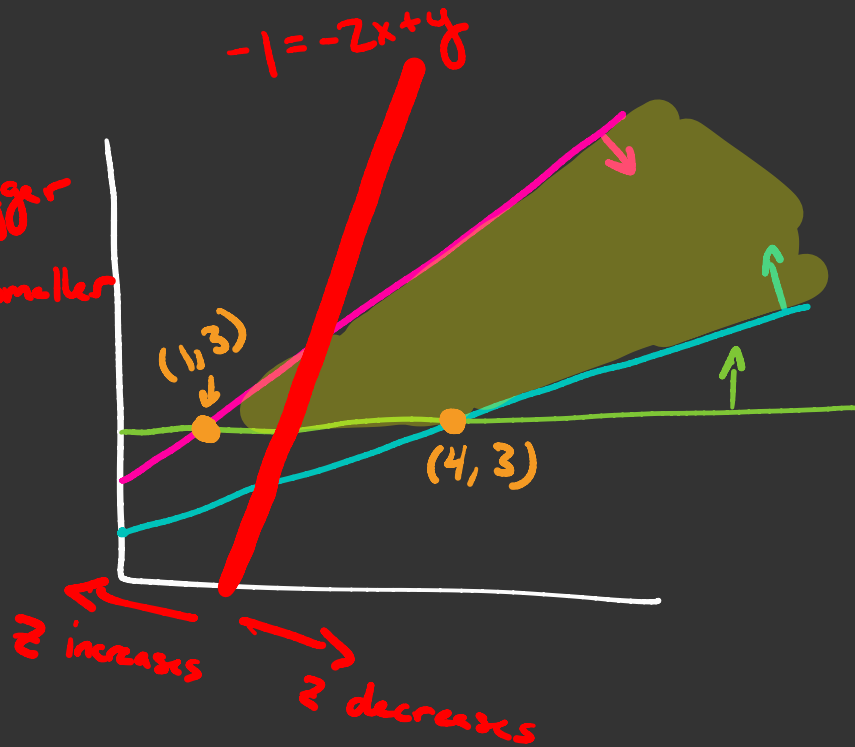
$z$  decreases  
 $z$  increases

$$b) z = -2x + y$$

$$(1, 3) : z = 1 \leftarrow \text{Bigger}$$

$$(4, 3) : z = -5 \leftarrow \text{Smaller}$$

Look at  $-1 = -2x + y$



Min: None

Max:  $z = 1$  at  $(1, 3)$

$$c) z = 2x + y$$

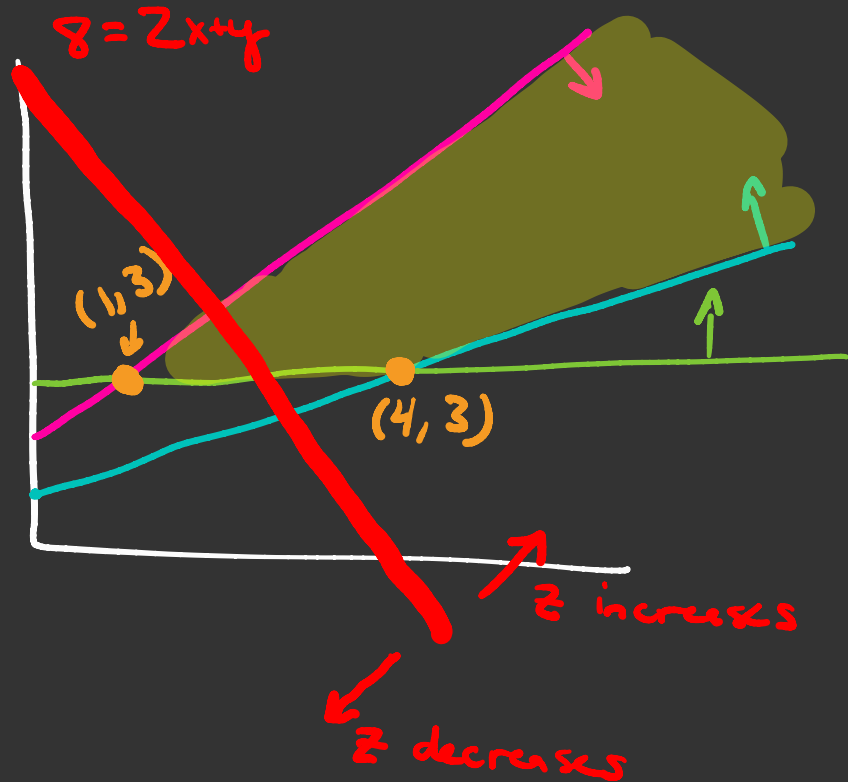
$$(1, 3): z = 5$$

$$(4, 3): z = 11$$

Look at  $z = 2x + y$

Min:  $z = 5$  at  $(1, 3)$

Max: None



# Matrices

Def: A matrix is a rectangular array of numbers

Example: 
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

↑  
size  $2 \times 3$   
→

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

↑  
size  $4 \times 3$

The size of a matrix is  $(\# \text{ rows}) \times (\# \text{ columns})$

We can define addition and multiplication with matrices

# Addition

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{bmatrix} = \begin{bmatrix} 1+10 & 2+11 \\ 3+12 & 4+13 \\ 5+14 & 6+15 \end{bmatrix}$$
$$= \begin{bmatrix} 11 & 13 \\ 15 & 17 \\ 19 & 21 \end{bmatrix}$$

Matrices must be same size to add!

Multiplication: Weird!

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 1 \times 10 + 2 \times 11 + 3 \times 12 \\ 4 \times 10 + 5 \times 11 + 6 \times 12 \end{bmatrix}$$

$(2 \times 3)$

$(3 \times 1)$

$(2 \times 1)$

Must match

$$= \begin{bmatrix} 68 \\ 167 \end{bmatrix}$$

Go across row of 1<sup>st</sup> matrix

Go along column of 2<sup>nd</sup> matrix

To multiply: # columns of 1<sup>st</sup> matrix = # rows 2<sup>nd</sup> matrix!

Example: a)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 4 + 2 \times 2 & 1 \times 3 + 2 \times 1 \\ 3 \times 4 + 4 \times 2 & 3 \times 3 + 4 \times 1 \end{bmatrix}$$

$(2 \times 2)$   $(2 \times 2)$   $(2 \times 2)$

match ✓

$$= \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix}$$



$$\begin{aligned} \text{b) } \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} &= \begin{bmatrix} 4 \times 1 + 3 \times 3 & 4 \times 2 + 3 \times 4 \\ 2 \times 1 + 1 \times 3 & 2 \times 2 + 1 \times 4 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 20 \\ 5 & 8 \end{bmatrix} \end{aligned}$$

Remark: It is not true in general that  $AB=BA$  for matrices  $A$  and  $B$