Challenge Problem: What would the principle of inclusion-exclusion look like for 3 sets?

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

What are some ways we can answer "How many...?" questions?

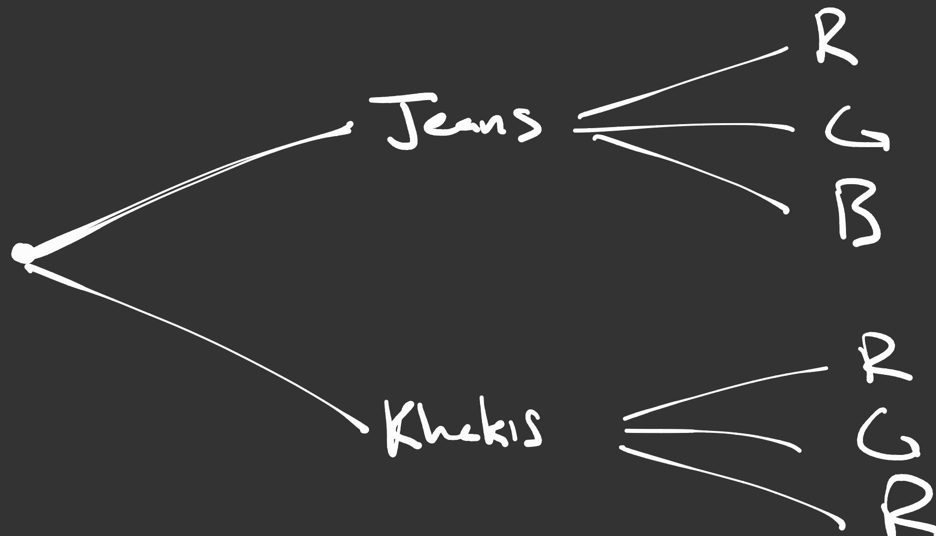
i) Listing all posibilities

Example: Getting dressed. Own 3 shirts and 2 pants

How many outfits?

1) Jeans, red shirt
2) Jeans, blue shirt
3) Jeans, green shirt
4) Khakis, red shirt
5) " , blue shirt
6) " green.

Helpful Tool: Tree Diagram



ii) Split activity into sequence of activities $A_1 \to A_2$

Example: A_1 = choosing pants (2 ways)

A_2 = choosing shirt (3 ways)

$\rightsquigarrow$ $2 \times 3 = 6$ ways to choose outfit

Multiplication Rule: If activities A_1, A_2 can be done in n_1, n_2 ways (resp.). Then # of ways to do A_1 followed by A_2 is $n_1 \times n_2$

Example: How many ways to deal out 2 cards to 2 different players (52 card deck)

A_1 = deal out 1^{st} card

A_2 = deal out 2^{nd} card

$$\rightsquigarrow \boxed{52 \times 51 = 2652 \text{ ways}}$$

iii) Break apart activity into separate cases

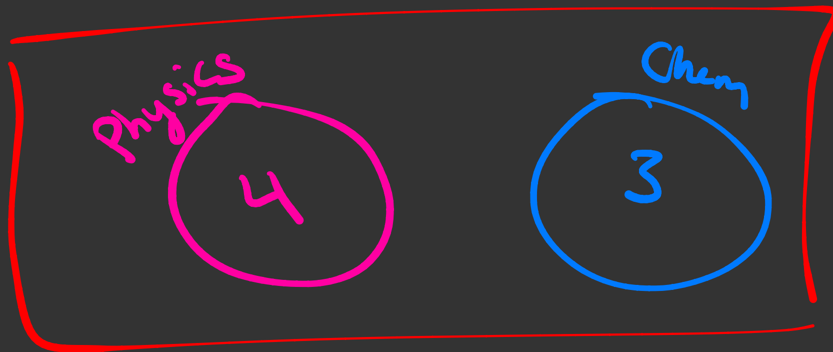
Example: Choose science credit. Physics or chemistry count

4 Physics classes

3 Chemistry

$\rightsquigarrow$ $4 + 3 = 7$ choices for science credit

Warning: It's important that the activities don't overlap. (Inclusion-Exclusion Priciple)



$$n(P\cup C) = n(P) + n(C) - \cancel{n(P\cap C)}^{\,0}$$

Addition Rule: Activity A_1 can be done in n_1 ways

" A_2 " " " " n_2 ways

$\rightsquigarrow$ # ways to do A_1 or A_2 (but not both) is

$$n_1 + n_2$$

Putting it all together: How many ways can we arrange 3 science books and 2 history books on a shelf, keeping subjects together?

$A_1 =$ science | history

$3 \times 2 \times 1 \times 2 \times 1 = 12$ ways

$A_2 =$ history | science

$2 \times 1 \times 3 \times 2 \times 1 = 12$ ways

$\boxed{24 \text{ ways}}$

Example: Suppose we have a keypad w/ numbers 1, 2, 3, 4, 5
Passcodes are 3 digits long

a) How many passcodes w/ different digits?

$\boxed{5 \times 4 \times 3}$

b) How many w/ repetition?

$\boxed{5 \times 5 \times 5}$

c) How many even passcodes w/ repetition? $\boxed{5 \times 5 \times 2}$

d) How many even passcodes w/o repetition? $3 \times 4 \times 2$

Choose digits backwards