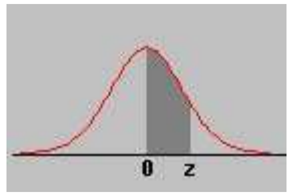
Standard Normal (Z) Table
Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Subtle point: The values in a normal distribution are continuous (no gaps)

$P(X=2)=0$

So, we talk about probabilities of intervals instead of points (like $P(1.5 \le X \le 2.3)$)

Challenge Problem: What are the mean/std dev of a binomial distribution w/

n trials
p prob of success
q prob of failure

For <u>one</u> trial: $E(X) = 0 \cdot q + 1 \cdot p = p$

X	$P(X)$
0	q
1	p

$$\sigma^2(X) = q(0-p)^2 + \underline{p(1-p)^2}$$

$$= qp^2 + pq^2 = pq(p+q) = pq$$

$$\sigma(X) = \sqrt{pq}$$

For 2 trials: $E(X) = p + p = 2p$

$$\sigma^2(X) = pq + pq = 2pq$$

$$\sigma(X) = \sqrt{2pq}$$

For n trials: $\boxed{\begin{aligned} E(X) &= np \\ \sigma^2(X) &= npq \\ \sigma(X) &= \sqrt{npq} \end{aligned}}$

Problem: As increases, computiting the probabilities in a binom distribution becomes harder and harder.

Solution: Approximate binomial distributions w/ normal distributions

Example: Use normal curve to approx prob of

a) 3 heads in 6 flips of coin

b) 3 or 4 heads in six flips

Binomial: a) $P_B(X=3) = C(6,3)(0.5)^3(0.5)^3$

$= \underline{0.3125}$

b) $P_B(3 \leq X \leq 4) = P_B(X=3) + P_B(X=4)$

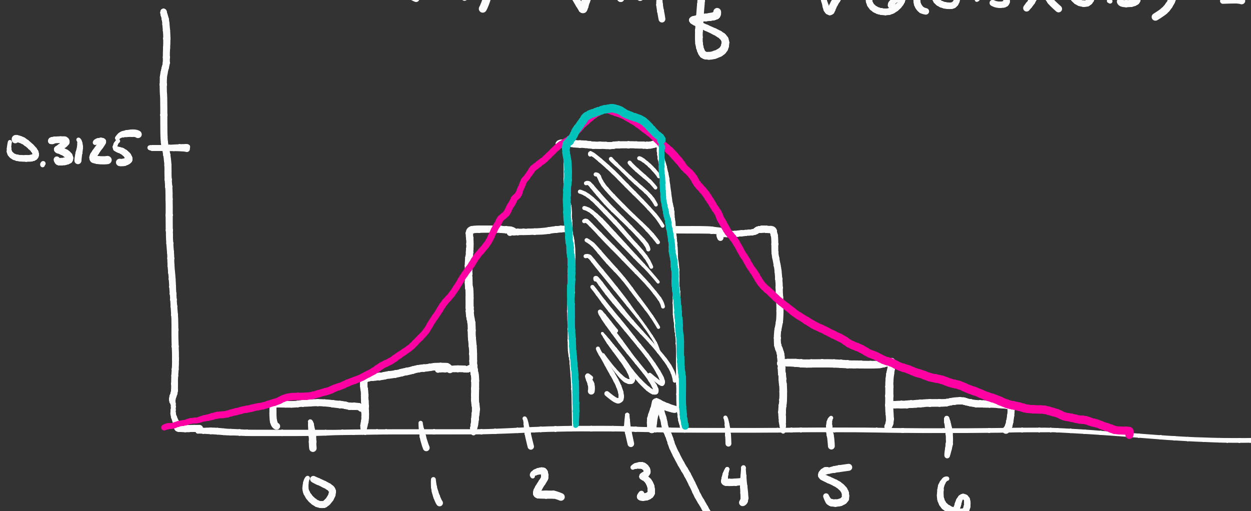
$= 0.3125 + C(6,4)(0.5)^4(0.5)^2$

$= 0.3125 + 0.2344$

$= 0.5469$

Normal: $\mu = E(X) = np = 6(0.5) = 3$

$$\sigma(X) = \sqrt{npq} = \sqrt{6(0.5)(0.5)} = 1.225$$



Want to approximate the area of bar

$$P_B(X=3) = \underline{P_N(2.5 \leq X \leq 3.5)}$$

$$P_N(2.5 \le X \le 3.5)$$

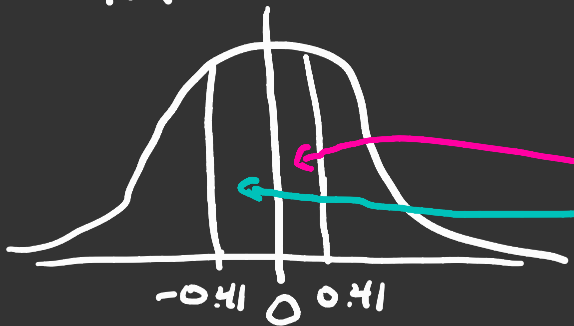
$$X = 2.5 \rightsquigarrow z = \frac{2.5-3}{1.225} = -0.41$$

$$X = 3.5 \rightsquigarrow z = \frac{3.5-3}{1.225} = 0.41$$

From normal dist table,

$$P_N(2.5 \le X \le 3.5) = P_N(-0.41 \le Z \le 0.41)$$

$$= 0.1591 + 0.1591 = 0.3182$$



b) $P_B(3 \le X \le 4) \approx P_N(\underbrace{2.5 \le X \le 4.5}_{\text{The x-values taken up by the 3-bar and 4-bar}})$

$$X = 4.5 \rightsquigarrow z = \frac{4.5-3}{1.225} = 1.22$$

$$P_N(2.5 \le X \le 4.5) = P_N(-0.41 \le z \le 1.22)$$

$$= 0.1591 + 0.3888 = 0.5479$$

Notation: $P_B \longrightarrow$ Binomial Probability (Area of bars in histogram)

$P_N \longrightarrow$ Area under normal curve

Rule: The normal distribution is a good estimate of the binom dist when both np and nq are greater than or equal to 5.