Example: Multiple choice quiz. 5 questions

4 possible choices/question

1 correct answer.

What is the prob we get exactly 3 questions right if we randomly guess?

✓✓✓XX: $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$

✓✓X✓X: $\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$

✓✓XX✓: $\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$

⋮

→ $C(5,3)$ different cases

$$\boxed{P(3\text{ correct}) = C(5,3)\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2}$$

This is an example of a <u>Bernoulli Experiment</u>

## Bernoulli Experiment w/ n trials

1. Repeated n times (n questions, n coin flips, etc)
2. Each trial has 2 outcomes: success / failure (right/wrong, H/T, ..)
3. Probability of success is same for each trial
4. Trials are independent
5. We are interested in number of successes, not order

Note: The championship example from is not a Bernoulli experiment. (could play 2 or 3 games)

Example: a) Coin flipped 8 times, count the number of heads

Is Bernoulli

b) 8 cards drawn from deck (without replacement), count number of spades

Not Bernoulli (trials not independent)

c) 8 cards drawn from deck (with replacement), count number of spades

Is Bernoulli

(with replacement)

d) Draw cards from deck until spade is drawn, count number of cards drawn.

**Not Bernoulli** (not repeated a fixed number of times)

Example: Roll die 3 times. What is prob of getting two 5's?

Bernoulli experiment? ✓

$$P(\text{Two 5's in 3 rolls}) = \underbrace{C(3,2)}_{\text{Ways to reorder 5's and non-5's.}}\underbrace{\left(\tfrac{1}{6}\right)^2}_{\text{Get two 5's}}\underbrace{\left(\tfrac{5}{6}\right)}_{\text{Get one non-5}}$$

$$= 3\left(\tfrac{1}{36}\right)\left(\tfrac{5}{6}\right) = 0.0694$$

# Probability of Bernoulli Experiment (Binomial)

$n$: repeated trials

$k$: number of success

$p$: probability of success

$q = 1-p$: probability of failure

$\rightsquigarrow$ $\underbrace{P(k \text{ successes in } n \text{ trials})}_{\text{sometimes written } P(X=k)} = C(n,k)\,p^k\, q^{n-k}$

The sequence of probabilities obtained from a Bernoulli experiment form the <u>binomial distribution</u>

Example: Roll die 3 times, count number of 5's.
Find binomial distribution for this experiment

| $X$ | $P(X)$ |
|---|---|
| 0 | $C(3,0)\left(\frac{5}{6}\right)^3 = 0.5787$ |
| 1 | $C(3,1)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2 = 0.3472$ |
| 2 | $C(3,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) = 0.0694$ |
| 3 | $C(3,3)\left(\frac{1}{6}\right)^3 = 0.0046$ |

Why is it called the binomial distribution?

Recall: Binomial Theorem

$$1 = (p+q)^3 = C(3,0)p^3q^0 + C(3,1)p^2q + C(3,2)pq^2 + C(3,3)p^0q^3$$

Probabilities of a binomial distribution