Example: Vending machine. Candy bar costs \$0.85

Put in a dollar. Three things can happen

- Get a candy bar and \$0.15 in change. Happens 80% of time
- Get a candy bar and no change. Happens 16% of time
- Get candy bar and our \$1 back. Happens 4% of time

What is average cost of candy bar?

Cost	Probability	Frequency (per 100)
\$0.85	0.8	80
\$1	0.16	16
\$0	0.04	4

$$\text{Mean cost} = \frac{80(0.85) + 16(1.00) + 4(0.00)}{100} = 0.84$$

$$= \frac{80}{100}(0.85) + \frac{16}{100}(1.00) + \frac{4}{100}(0.00)$$

Expected Value

$$= \boxed{0.8(0.85) + 0.16(1.00) + 0.04(0.00)}$$

Definition: If a random variable X has values $x_1, x_2, \ldots, x_n$ with corresponding probabilities $p_1, p_2, \ldots, p_n$, then the expected value of X is

$$E(X) = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

Example: Coin flip game X = amount won

a) Heads $\longrightarrow$ Win \$5
Tails $\longrightarrow$ Win \$1

$$E(X) = \frac{1}{2}(5) + \frac{1}{2}(1) = 3$$

b) Heads $\longrightarrow$ Win \$4
Tails $\longrightarrow$ Lose \$3

$$E(X) = \frac{1}{2}(4) + \frac{1}{2}(-3) = 0.5$$

c) Heads $\rightsquigarrow$ Win \$5

Tails $\rightsquigarrow$ Lose \$5

$$E(X) = \tfrac{1}{2}(5) + \tfrac{1}{2}(-5) = 0$$

Game is perfectly fair (favors neither person)

Example: Roulette. 38 numbers

Put \$1 on number 4.

Lands on 4 $\rightsquigarrow$ Win \$35

Anything else $\longrightarrow$ Lose \$1

$$E(X) = \frac{1}{38}(35) + \frac{37}{38}(-1) = \frac{-2}{38} \approx -0.05$$

Not in our favor

Example: Lottery. 10000 tickets, 1 wins
9999 lose

Winner gets \$1 million, losers get nothing

How much should they charge to make this fair?

c = cost of ticket.

$$E(X) = 0.0001(1000000 - c) + 0.9999(0 - c)$$

$$= 100 - 0.0001c - 0.9999c$$

$$= 100 - c = 0$$ If game is fair

$$\rightsquigarrow c = 100$$

Variance / Standard Dev of Random Variables

Cost	Probability	Frequency (per 100)
\$0.85	0.8	80
\$1	0.16	16
\$0	0.04	4

Mean = 0.84

$$\text{Variance} = \frac{80(0.85-0.84)^2 + 16(1-0.84)^2 + 4(0-0.84)^2}{100}$$

$$= 0.8(0.85-0.84)^2 + 0.16(1-0.84) + 0.04(0-0.84)^2$$

Definition: If X is a random variable with values $x_1, x_2, \ldots, x_n$, corresponding probs $p_1, p_2, \ldots, p_n$, and expected value $E(X) = \mu$, then

$$\text{Variance} = \sigma^2(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \cdots + p_n(x_n-\mu)^2$$

$$\text{Standard Dev} = \sigma(X) = \sqrt{\text{Variance}}$$

Example:

x_i	p_i
4	0.2
7	0.2
10	0.5
8	0.1

$\sigma(X) = \sqrt{5.4}$

$= 2.32$

x_i	p_i	$p_i x_i$	$x_i - \mu$	$(x_i - \mu)^2$	$p_i(x_i - \mu)^2$
4	0.2	0.8	-4	16	3.2
7	0.2	1.4	-1	1	0.2
10	0.5	5	2	4	2.0
8	0.1	0.8 +	0	0	0 +
		8.0			5.4
		$\mu = E(X)$			$\sigma^2(X)$