Department of Mathematics University of Notre Dame Math 10120 – Finite Math Spring 2020

Name:		

Instructor: Juan Migliore

Exam 2

March 5, 2020

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

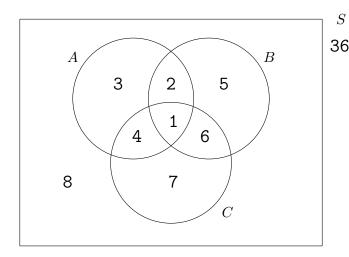
The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an \times through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
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5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B) and Colombia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability that she is going to exactly two of the countries (not one, not three)?



- (a) $\frac{15}{36}$
- (b) $\frac{13}{36}$
- (c) $\frac{12}{36}$
- (d) $\frac{27}{36}$
- (e) $\frac{1}{36}$

- **2.** (5 pts.) Referring to the Venn diagram in problem #1, find P(A'|B).
- (a) $\frac{3}{14}$
- (b) $\frac{11}{36}$
- (c) $\frac{11}{26}$
- (d) $\frac{5}{14}$
- (e) $\frac{11}{14}$

3. (5 pts.) Suppose A, B, C are three events with the following probability information:

$$P(A) = \frac{1}{2},$$
 $P(B) = \frac{1}{3},$ $P(C) = \frac{1}{4}$

$$P(B) = \frac{1}{2}$$

$$P(C) = \frac{1}{4}$$

$$P(A \cap B) = \frac{5}{6}, \qquad P(A \cap C) = \frac{1}{2}, \qquad P(B \cap C) = \frac{1}{12}$$

$$P(A \cap C) = \frac{1}{2}$$

$$P(B \cap C) = \frac{1}{12}$$

Which pairs of events are independent?

- (a) B and C are independent
- (b) A and B are independent
- A and C are independent (c)
- A and C are independent, and also B and C are independent (d)
- A and B are independent, and also A and C are independent (e)

4. (5 pts.) The Dakota Club has six members from North Dakota and two members from South Dakota. Two members are chosen at random to represent the club at a national conference. What is the probability that the two chosen members are from the same state?

- (b) $\frac{15}{28}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$ (e) $\frac{3}{4}$

5. (5 pts.) In a certain town in Canada, 50% of the population speaks English, 70% speaks French and 30% speaks both English and French. A person is chosen at random, and it is noted that she speaks English. What is the probability that she also speaks French?

- (a) 70%
- (b) 60%
- (c) 30%
- (d) 40%
- (e) 20%

6. (5 pts.) Claire belongs to a certain club that has a total of 7 members. The club is supposed to choose a team of three people to represent the club in a national conference. If the three are chosen at random, what is the probability that Claire is one of the three chosen?

- (a) $\frac{1}{7}$
- (b) $\frac{1}{3}$
- (c) $\frac{3}{35}$
- (d) $\frac{1}{35}$
- (e) $\frac{3}{7}$

 $P(\frac{\text{Claire is}}{\text{chosm}}) = \frac{C(6.2)}{C(7.3)} = \frac{\frac{6!}{2!4!}}{\frac{7!}{3!4!}} = \frac{6!}{7.6.5}$

7. (5 pts.) On a certain large plane, 8 passengers are sitting in row 52. Of these, 6 chose broccoli for their vegetable at supper and 2 chose cauliflower. Of the broccoli passengers, 4 watched a movie after supper and 2 took a nap. Of the cauliflower passengers, both took naps after supper. The flight crew randomly chose a passenger from row 52 to ask about the flight, and learned that that passenger took a nap after supper. What is the probability that that passenger also had broccoli?

- (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$ (e) $\frac{1}{4}$

8. (5 pts.) Emily is given a fair coin and told that she can flip it up to four times. She stops when it lands heads, or after the fourth flip. She wins if it ever lands heads. Otherwise she loses. What is the probability that she wins?

- (a)
- (b) $\frac{3}{4}$ (c)

- $\frac{15}{16}$ (d) $\frac{1}{2}$ (e) $\frac{1}{16}$

9. (5 pts.) I have three boxes. Box I has 3 blue balls and 0 red ones. Box II has 2 blue balls and 2 red ones. Box III has 6 blue balls and 2 red ones. I choose a box at random and draw a ball from it. If the ball I draw is blue, what is the probability that it came from box III?

- (b) $\frac{6}{11}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
- (e)

10. (5 pts.) You are dealt 5 cards from a standard deck of 52 cards (without replacement). What is the probability that you get a full house (i.e. what is the probability that you get three of one rank and two of another; e.g. three kings and two 3's)? Remember that there are 13 ranks: A,2,3,4,5,6,7,8,9,10,J,Q,K.

(c) $\frac{13 \cdot C(4,3) \cdot 12 \cdot C(4,2)}{C(52,5)}$

- $\begin{array}{lll} \text{(a)} & \frac{13 \cdot C(4,3) \cdot C(4,2)}{C(52,5)} & \text{(b)} & \frac{C(13,2)}{C(52,5)} \\ \\ \text{(d)} & \frac{C(4,3) \cdot C(4,2)}{C(52,5)} & \text{(e)} & \frac{13 \cdot 12 \cdot C(52,3) \cdot C(52,2)}{C(52,5)} \end{array}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations (P(n,k)), combinations (C(n,k)), factorials (n!) and powers (a^k) . Be sure to mark your answer.

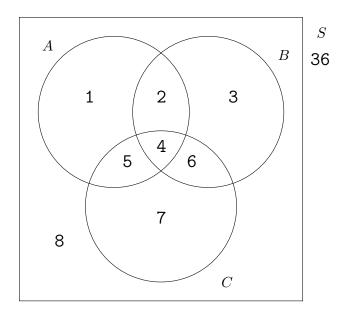
Amy, Bernie, Elizabeth, Joe, Mike, Pete and Tom are running in a race. Assume that they are equally likely to win, and that there can be no ties.

(a) What is the probability that the girls (Amy and Elizabeth) both finish before all the boys. [Remember to take into account whether Amy is first and Elizabeth second, or vice versa.]

Answer to (a):
$$\frac{2 \cdot 5!}{7!} = \frac{2 \cdot P(5,5)}{P(7,7)}$$

(b) What is the probability that the seven runners finish in alphabetical order (i.e. in the order stated above)?

12. (10 pts.) The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



Find each of the probabilities using the numbers in the diagram. For example, if it asked for P(A) you could write any one of the following:

$$\frac{1}{3}$$
 or $\frac{12}{36}$ or $\frac{1+2+4+5}{36}$.

(a)
$$P(A \cup B) =$$

(b)
$$P(B \mid A) =$$

(c)
$$P(A \cap B' \cap C) =$$

(d)
$$P(A \cap B \mid C) =$$

(e)
$$P(A \cap B \mid C') =$$

13. (10 pts.) One quarter (1/4) of the people in a certain city prefer to read physical books (B) while three quarters (3/4) prefer to read some sort of tablet (T).

- Of the people who prefer physical books, four fifths (4/5) are over 60 years old (OS) and one fifth (1/5) are under 60 (US).
- Of the people who prefer tablets, one third (1/3) are over 60 (OS) and two thirds (2/3) are under 60 (US).
- (a) Draw a tree diagram representing this situation.

(b) If a person in that city is chosen at random, find the probability that he/she is over 60.

(c) A person in that city is chosen at random, and it is determined that he/she is over 60. Find the probability that he prefers to read physical books. Show your work!!!

14. (10 pts.) There are 6 students in a certain seminar, and each is asked for his/her birth month. Assume for simplicity that all months are equally likely, so the only information you need is that there are 12 months.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations (P(n,k)), combinations (C(n,k)), factorials (n!) and powers (a^k) . Be sure to mark your answer.

(a) (6 pts.) What is the probability that all 6 of the students were born in **different** months?

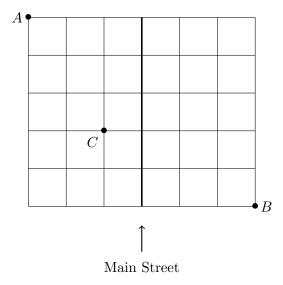
Answer to (a):

(b) (4 pts.) What is the probability that **at least** two of the students were born in the same month? [Hint: use your answer to (a).]

Answer to (b):

15. (10 pts.) **Note:** In this problem, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations (P(n,k)), combinations (C(n,k)), factorials (n!) and powers (a^k) . Be sure to show all your work and be sure to plainly mark your answer.

The following is a street map of part of a city. Aaron starts at corner A, and is planning to go all the way to Bob's house at B. At each intersection he only goes east (right) or south (down). He makes that choice randomly unless there is only one choice. (E.g. at the top rightmost corner he only has one choice, namely south.) Main Street is the street marked in boldface.



- (a) How many routes are there to get to Bob's house? (Main Street plays no role in this part.)
- (b) Claire lives at the intersection marked C. What is the probability that Aaron's route passes by Claire's house? (Main Street plays no role in this problem either.)
- (c) What is the probability that Aaron's route includes all five blocks of Main Street? [Hint: how many routes include all five blocks of Main Street?]

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Practice Exam 2

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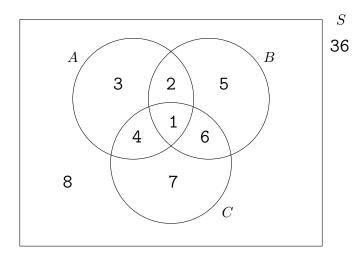
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Multiple Choice

1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B) and Colombia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability that she is going to Argentina?



- (b)
- (d)

2. (5 pts.) Refer to the club and Venn diagram in problem #1. A student is chosen at random, and it is discovered that he is going to Argentina. With this extra information, what is the probability that he is also going to Bolivia?

- $\frac{3}{10}$ (b) $\frac{2}{10}$ (c) $\frac{3}{14}$ (d) $\frac{2}{14}$ (e)

3. (5 pts.) Claire rolls a red die and a blue die and observes the sum. Find the probability that the sum is odd. [Note: "odd" means, in this case, either 3, 5, 7, 9 or 11.]

(a) $\frac{12}{36} = \frac{1}{3}$

(b) $\frac{9}{36} = \frac{1}{4}$

(c) $\frac{5}{11}$

(d) $\frac{6}{36} = \frac{1}{6}$

(e) $\frac{18}{36} = \frac{1}{2}$

4. (5 pts.) Emily flips a coin 6 times. Find the probability that the coin shows Heads exactly 3 times. Give your answer as a fraction in lowest terms.

(a)
$$\frac{1}{2}$$

(b)
$$\frac{1}{8}$$

(c)
$$\frac{5}{16}$$

$$(\mathrm{d}) \quad \frac{1}{20}$$

(e)
$$\frac{3}{16}$$

$$=\frac{C(6,3)}{10}$$

$$=\frac{20}{2^{6}}=\frac{5}{2^{4}}$$

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 $\mathbf{5.}$ (5 pts.) In a certain town in Canada, 50% of the population speaks English, 70% speaks French and 10% doesn't speak either English or French. A person is chosen at random. What is the probability that she speaks both English and French?

- (a) 90%
- (b) 10%
- (c) 20%
- (d) 30%
- (e) 60%

6. (5 pts.) The Smith family has 10 children, consisting of four boys and six girls. After a big snowstorm, the parents randomly choose three of the children to shovel the driveway. What is the probability that at least one of the three is a boy?

- (a) $\frac{5}{6}$
- (b) $\frac{1}{6}$
- (c) $\frac{1}{3}$
- (d) $\frac{1}{2}$
- (e) $\frac{2}{3}$

7. (5 pts.) In a certain city, 55% of the people are male and 45% are female. Of the males, 15% have a certain disorder. Of the females, 5% have the disorder. A person is chosen at random and found **not** to have the disorder. What is the probability that **that** person is female? [Hint: start with a tree diagram.]

- (a) $\frac{(.45)(.95)}{(.45)(.95) + (.45)(.05)}$
- (b) $\frac{(.45)(.95)}{(.55)(.85)}$

(c) .45

(d) (.45)(.95)

(e) $\frac{(.45)(.95)}{(.45)(.95) + (.55)(.85)}$

8. (5 pts.) In randomly chosen group of 15 people, what is the probability that **at least** two have the same birthday?

(a) $\frac{C(15,2)}{365^{15}}$

(b) $\frac{C(15,2)(363)(362)\cdots(351)}{365^{15}}$

(c) $1 - \frac{(365)(364)\cdots(351)}{365^{15}}$

(d) $1 - \frac{C(15,2)}{365^{15}}$

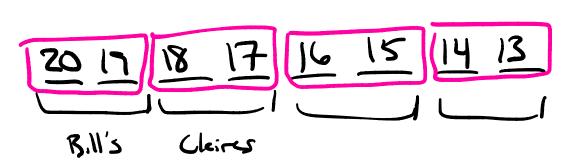
- (e) $1 \frac{C(15,2) \cdot C(363,2)}{365^{15}}$
- $P(\frac{\text{st bust}}{2 \text{ share B-day}}) = 1 P(\frac{\text{everyone has}}{4 \cdot \text{ff B-day}})$ $= 1 \frac{P(365.15)}{365.15}$

3LS 3LH 3L3 362 3L

9. (5 pts.) I have a deck of 24 cards consisting of the A, 2, 3, 4, 5 and 6 of each suit. Bill, Claire, Doug and Emily are seated around a table. From this deck the dealer gives 2 cards to Bill, two cards to Claire, two cards to Doug and two cards to Emily (**without replacement**). What is the probability that **none** of them gets an ace? [Hint: of the 24 cards, four are aces and 20 aren't.]

- (a) $P(20,2) \cdot P(18,2) \cdot P(16,2) \cdot P(14,2)$ P(24,8)
- (b) $1 \frac{P(20,8)}{P(24,8)}$
- (c) $\frac{P(24,2) \cdot P(22,2) \cdot P(20,2) \cdot P(18,2)}{P(24,8)}$
- (d) $1 \frac{4 \cdot P(20, 2)}{P(24, 8)}$

(e) $\frac{4 \cdot P(20, 2)}{P(24, 8)}$



P(20,8)
P(24,8)

10. (5 pts.) You are dealt 13 cards from a standard deck of 52 cards (without replacement). What is the probability that you get both the Jack of diamonds and the Queen of spades?

(a) $\frac{C(52,11)}{C(52,13)}$

(b) $\frac{P(50,11)}{C(52,13)}$

(c) $\frac{C(13,2) \cdot C(39,11)}{C(52,13)}$

(d) $\frac{C(50,11)}{C(52,13)}$

(e) $\frac{2 \cdot C(50, 11)}{C(52, 13)}$

$$P(And Q \Omega) = \frac{((50,11))}{((52,13))}$$

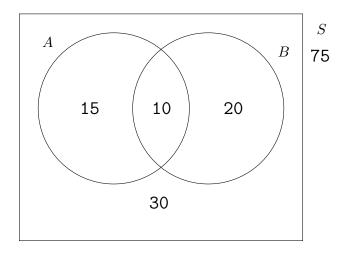
C(50,11)

Partial Credit

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11. (10 pts.)

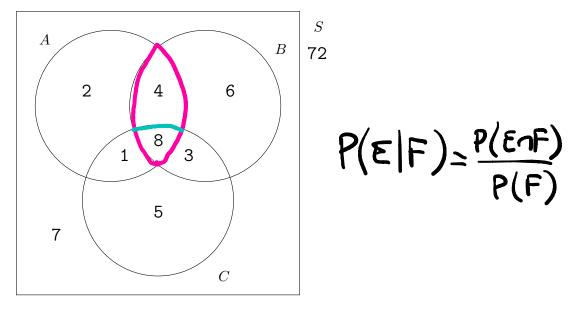
The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



(a) Show that events A and B are independent. You have to fully explain your answer to get credit.

(b) Are events A and B mutually exclusive? Give a clear explanation of your answer.

12. (10 pts.) The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



Find each of the probabilities using the numbers in the diagram. For example, if I asked for P(A), I'd like you to write

$$\frac{2+4+8+1}{72}$$
. You would not have to write $=\frac{15}{72}=\frac{5}{24}$ unless you want to.

(Any order of the numbers in the numerator would be fine.)

(a)
$$P(A \cap B) = \frac{8+4}{72}$$

(b)
$$P(C \mid A) = \frac{n(C \cap A)}{n(A)} = \frac{1+8}{2+4+1+8}$$

(c)
$$P(A \cap B \cap C') = \frac{\mathcal{H}}{\mathcal{F}Z}$$

(d)
$$P(A \cap B \mid C) = \frac{n(A \cap B \cap C)}{n(C)} = \frac{8}{1+8+1}$$

(e)
$$P(C \mid A \cap B) = \frac{n(Cn(A \cap B))}{n(A \cap B)} = \frac{8}{8+4}$$

13. (10 pts.) (These numbers are totally fictitious.) In the city of Philadelphia, 99% of the people are Eagles fans and 1% are not.

- \bullet Of the Eagles fans, 98% own an Eagles sweatshirt and 2% do not.
- Of the non-Eagles fans, 5% own an Eagles sweatshirt and 95% do not.
- (a) Draw a tree diagram representing this situation. Explain your notation and be sure to include the probabilities.

(b) If a person in Philadelphia is chosen at random, find the probability that she owns an Eagles sweatshirt. Show your work!!!

(c) A person in Philadelphia is chosen at random, and it is determined that he does not own an Eagles sweatshirt. Find the probability that he is an Eagles fan. Show your work!!!

14. (10 pts.) A bag contains 9 colored marbles, of which 5 are red and 4 are blue. I plan to pick 3 marbles from the bag **without replacement**.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations (P(n,k)), combinations (C(n,k)), factorials (n!) and powers (a^k) . Be sure to mark your answer.

(a) Assuming that order is not important, what is the probability that all three are the same color?

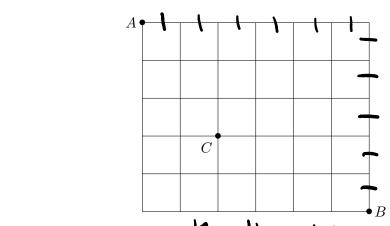
(b) Assuming that order **is** important, what is the probability that the first is red, then the second is blue, then the third is red. [Don't forget that this is done **without** replacement.]

15. (10 pts.) In this problem, be sure to show all your work and be sure to plainly mark your answer.

The following is a street map of part of a city. Emily lives at the northwest corner (marked A) and wants to get to the library at the southeast corner (marked B). She decides to take an Uber. The driver travels only east and south (i.e. to the right or down), following the roads and randomly choosing between east and south at each intersection he comes to. What is the probability that the Uber will pass by Claire's house, marked C?

[Hint: how many routes are there to get to the library, and of those how many pass by \mathbb{C} ?]

Note: In this problem, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations (P(n,k)), combinations (C(n,k)), factorials (n!) and powers (a^k) .



n(C)

C(5,2).C(6,4)

C(11,6)

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