

Types of averages:

- Mean ← Most common
- Median
- Mode

The mean of  $n$  numbers  $x_1, x_2, \dots, x_n$  is given by

$$\mu = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Example: The mean of 2, 3, 5, 7, 9 is

$$\frac{2+3+5+7+9}{5} = \frac{26}{5} = 5.2$$

Example: Scores on golf hole in tournament

Score	Frequencies	Rel Freq
2	1	$1/50 = 0.02$
3	15	$15/50 = 0.3$
4	21	$21/50 = 0.42$
5	11	$11/50 = 0.22$
6	2	$2/50 = 0.04$

$$\begin{aligned}\text{Mean Score} &= \frac{1(2) + 15(3) + 21(4) + 11(5) + 2(6)}{50} = \frac{198}{50} = 3.96 \\ &= 0.02(2) + 0.3(3) + 0.42(4) + 0.22(5) + 0.04(6)\end{aligned}$$

# Example: Test Score

Scores	Frequency
50-59	1
60-69	2
70-79	3
80-89	5
90-100	4

Can't compute the mean exactly, but can approximate

Midpoint of 50-59

Midpoint of 60-69

$$\begin{aligned}\text{Mean Score} &= \frac{1(54.5) + 2(64.5) + 3(74.5) + 5(84.5) + 4(95)}{15} \\ &= 80.63\end{aligned}$$

Example: Class w/ 3 juniors, 2 seniors

On exam, juniors averaged 80% and  
seniors averaged 90%

$$\frac{x_1 + x_2 + x_3}{3} = 80$$

$$\frac{y_1 + y_2}{2} = 90$$

What is the class average?

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + y_1 + y_2}{5} = \frac{x_1 + x_2 + x_3}{5} + \frac{y_1 + y_2}{5}$$

$$= \underbrace{\frac{x_1 + x_2 + x_3}{3}}_{80} \left( \frac{3}{5} \right) + \underbrace{\frac{y_1 + y_2}{2}}_{90} \left( \frac{2}{5} \right)$$

$$= 80 \left( \frac{3}{5} \right) + 90 \left( \frac{2}{5} \right) = \frac{80(3) + 90(2)}{5}$$

# Example: Salaries at company

Position	# people in pos	Salary
Pres	1	300,000
VP	1	40,000
Production Workers	15	25,000

$$\text{Mean Salary} = \frac{300000 + 40000 + 15(25000)}{17} \approx \underline{\underline{42,060}}$$

Problem: Extreme values can throw off mean

Higher than all but one person's salary!

The median is the middle number after the numbers have been arranged in order

Example: ~~57~~, ~~64~~, ~~68~~, 71, ~~73~~, ~~79~~, ~~80~~

$$\text{Median} = 71$$

Example: ~~57~~, ~~64~~, 68, 71, ~~73~~, ~~79~~

$$\text{Median} = \frac{68+71}{2} = \frac{139}{2} = 69.5$$

Intuition: Median essentially divides the data points in half

Example:

Position	# people in pos	Salary
Pres	1	300,000
VP	1	40,000
Production Workers	15	25,000

Median = 25000

The mode(s) of a data set is the value that occurs the most often (can be more than one value)

Example: 1, 1, 1, 2, 3, 4, 4, 5, 5, 5, 6, 7, 8, 8

Mode = 1, 5

Example: Mode Salary = 25,000



Which average is best? It depends

Mean: • No extreme values  
• Equal distances between scores mean  
equal distances between things being  
measured

Median: • Extreme values  
• Rankings

Mode: • Things like clothing sizes  
(need to know values  
occurring most often)