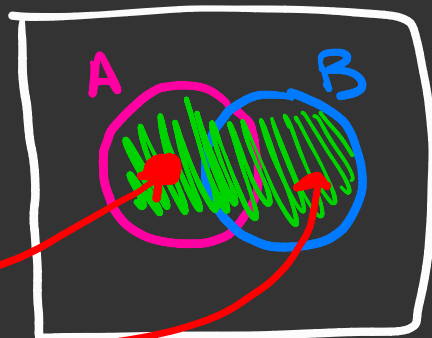
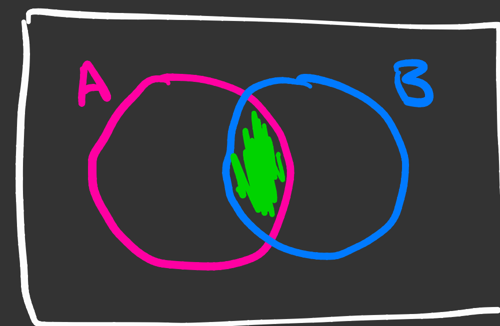
Challenge Problem: When does $A \cup B = A \cap B$?

can't be elements here



$A \cup B$



$A \cap B$

$\rightsquigarrow A = B$

Notation: $n(A) = \#$ of elements in A

(Sometimes you'll see $|A|$)

Question: If we know $n(A)$ and $n(B)$, do we know $n(A \cup B)$?

$$\{1,2,3\} \cup \{4,5,6\} = \{1,2,3,4,5,6\}$$

$$\{1,2,3\} \cup \{2,3,4\} = \{1,2,3,4\}$$

Activity: A = students in class w/ birthdays Jan-Sept

$n(A) = 13$

B = students in class w/ birthdays Apr-Dec

$n(B) = 13$

$$19 = n(A \cup B) \cancel{=} n(A) + n(B) = 13 + 13$$

$$19 = \boxed{n(A \cup B) = \underline{n(A)} + \underline{n(B)} - n(A \cap B)} = 13 + 13 - 7$$

Principle of Inclusion-Exclusion

Double count people in $A \cap B$

Example: Suppose $n(A) = 18$, $n(B) = 6$, $n(A \cup B) = 20$

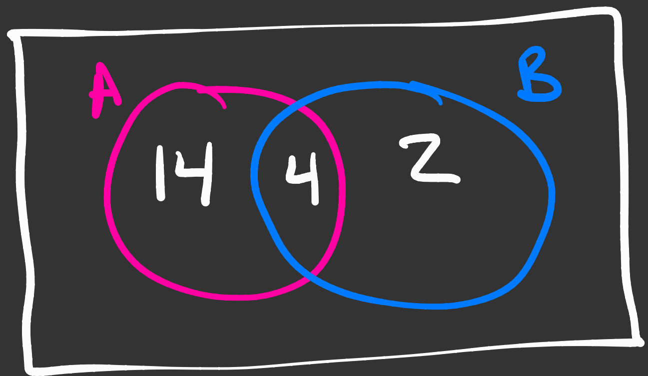
Find $n(A \cap B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

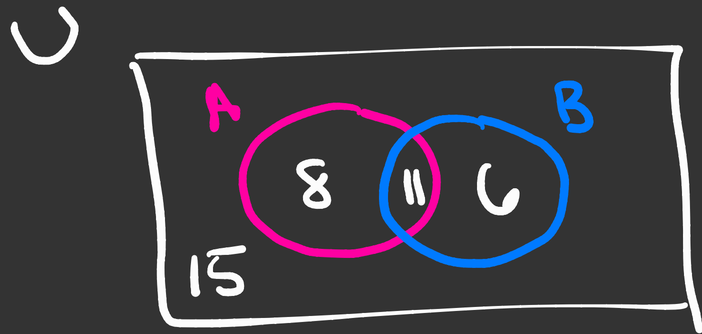
$$20 = 18 + 6 - n(A \cap B)$$

$$= 24 - n(A \cap B)$$

$$\boxed{n(A \cap B) = 4}$$



Example: Suppose two sets have the following Venn Diagram.



$$n(A) = 19$$

$$n(B) = 17$$

$$n(A \cup B) = 8 + 11 + 6 = 25$$

$$n(A') = 6 + 15 = 21$$

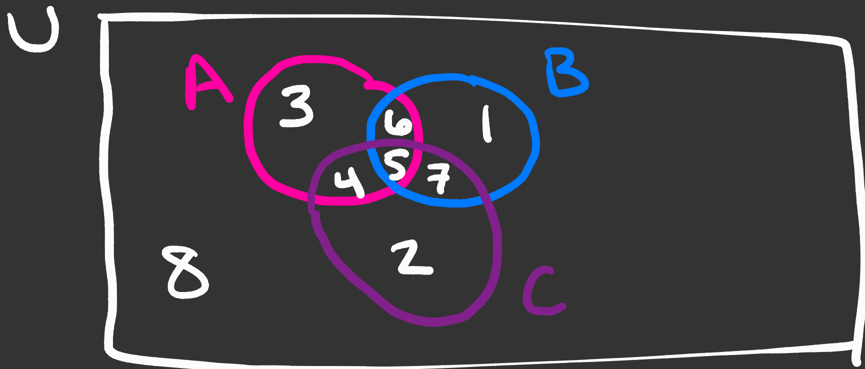
$$n(B') = 8 + 15 = 23$$

$$n(A \cup B') = 8 + 11 + 15 = 34$$

$$n((A \cup B)') = 15$$

$$n((A \cap B)') = 15 + 8 + 6 = 29$$

Venn Diagrams of 3 sets



$$n(A) = 3+4+5+6 = 18$$

$$n(A \cap B) = 6+5 = 11$$

$$n(A \cap B \cap C) = 5$$

$$n((A \cup B)') = 8+2 = 10$$

Fun Fact: $A' \cup B' = (A \cap B)'$

$$\rightarrow n(A' \cup B') = 8+2+7+1+4+3 = 25$$

$$\rightarrow n((A \cap B)') = 25$$

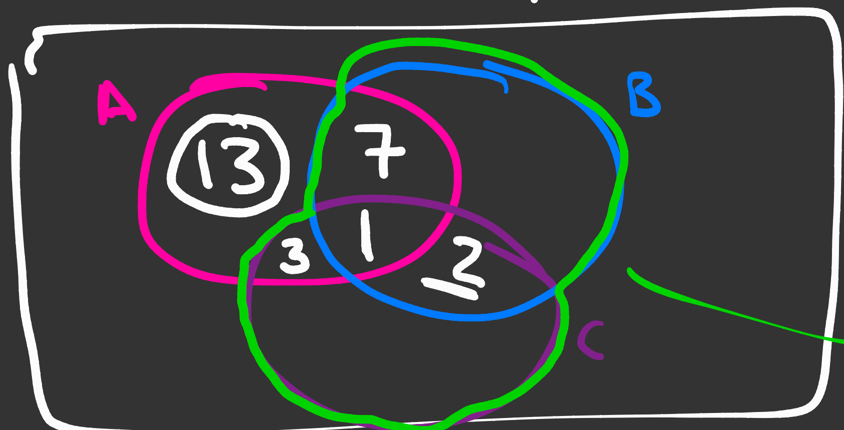
$$n(A \cup (B \cap C)) = 3+4+5+6+7 = 25$$

Example: Suppose $n(A \cup B \cup C) = 48$

$n(A) = 24 \quad n(B) = 22 \quad n(C) = 16$

$n(A \cap B) = 8 \quad n(A \cap C) = 4$

$n(A \cap B \cap C) = 1$



Rule of thumb: Start from the smallest set $(A \cap B \cap C)$

$\rightarrow n(B \cup C) = 35$

$35 = n(B \cup C) = n(B) + n(C) + n(B \cap C) = 22 + 16 - n(B \cap C)$

$= 38 - n(B \cap C)$

$n(B \cap C) = 3$