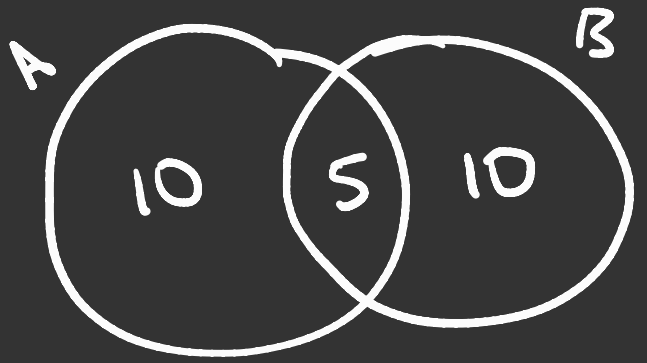
Challenge Problem: Are $P(A|B)$ and $P(B|A)$ equal?



$$P(B|A) = \frac{5}{15}$$

$$P(A|B) = \frac{5}{15}$$

- One way: $P(A) = P(B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

- Second way: A and B mut exclusive
- Not equal when. A = Student gets A, B = student passes class

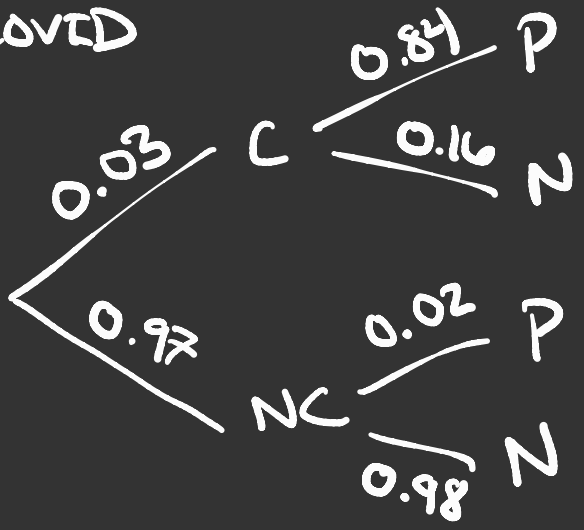
Example: COVID Testing

C = person has COVID

NC = person doesn't have COVID

P = test positive

N = test negative



$$P(P) = P(C \cap P) + P(NC \cap P)$$

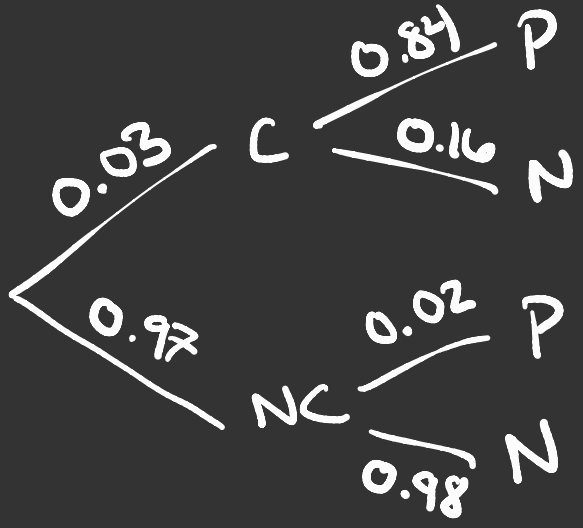
$$= 0.03 \times 0.84 + 0.97 \times 0.02$$

$$= 0.0446$$

$$P(N) = 1 - P(P)$$

$$= 0.9554$$

$$P(N \mid C) = 0.16$$



$$P(NC|P) = \frac{P(NC \cap P)}{P(P)}$$

$$= \frac{0.97 \times 0.02}{P(C \cap P) + P(NC \cap P)}$$

$$= \frac{0.97 \times 0.02}{0.0446}$$

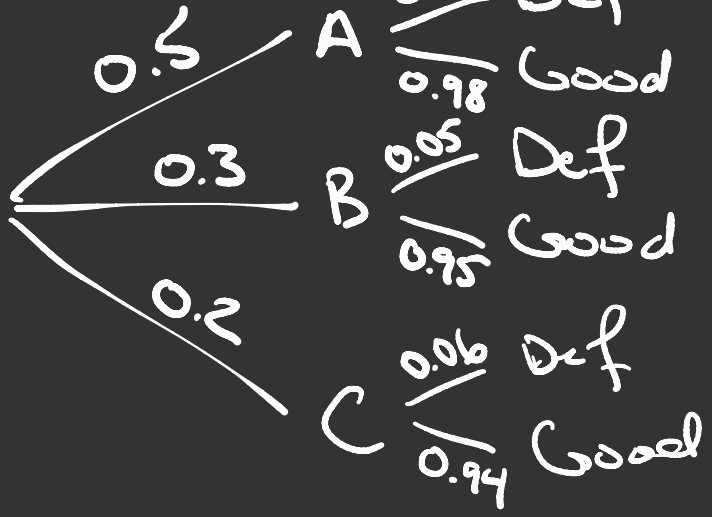
$$= 0.435$$

$$P(C|P) = 1 - 0.435 = 0.565$$

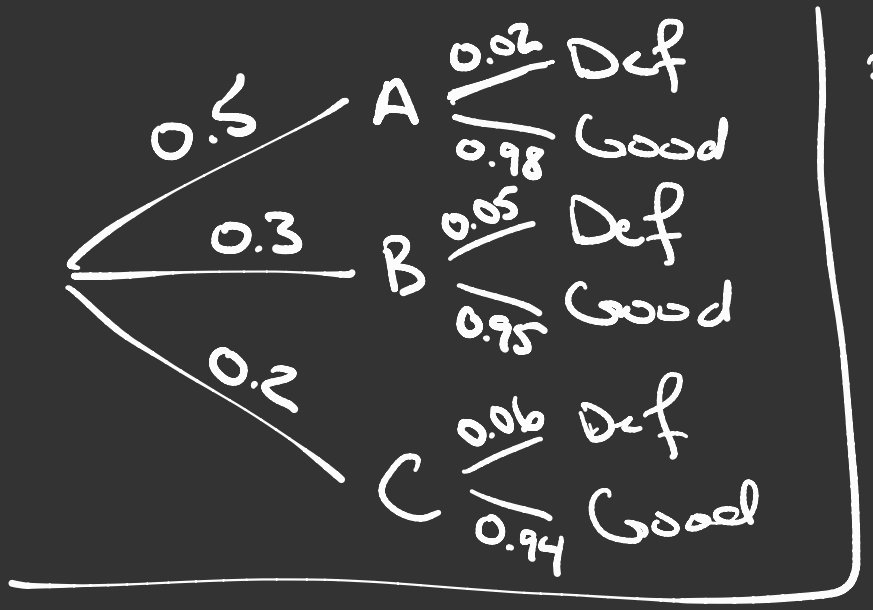
Challenge: Compute probability of false negative.

Example: Sourcing a part from 3 manufacturers: A, B, C

- Get 50% of parts from A, 2% defective
- 30% B, 5% def
- 20% C, 6% def



Suppose we have a bad part. Which manufacturer is probably to blame?



$$P(A|Def) = \frac{P(A \cap Def)}{P(Def)}$$

$$= \frac{0.5 \times 0.02}{P(A \cap Def) + P(B \cap Def) + P(C \cap Def)}$$

$$= \frac{0.5 \times 0.02}{0.5 \times 0.02 + 0.3 \times 0.05 + 0.2 \times 0.06}$$

$$= \frac{0.01}{0.037} = 0.27$$

$$P(B|Def) = \frac{P(B \cap Def)}{P(Def)} = \frac{0.3 \times 0.05}{0.037} = 0.41$$ ← B is probably to blame!

$$P(C|Def) = 1 - P(A|Def) - P(B|Def) = 1 - 0.27 - 0.41$$

$$= 0.32$$

Are the events A and Def independent?

- $P(A \cap Def) \stackrel{?}{=} P(A) \cdot P(Def)$

  $P(A \cap Def) = 0.5 \times 0.02 = 0.01$

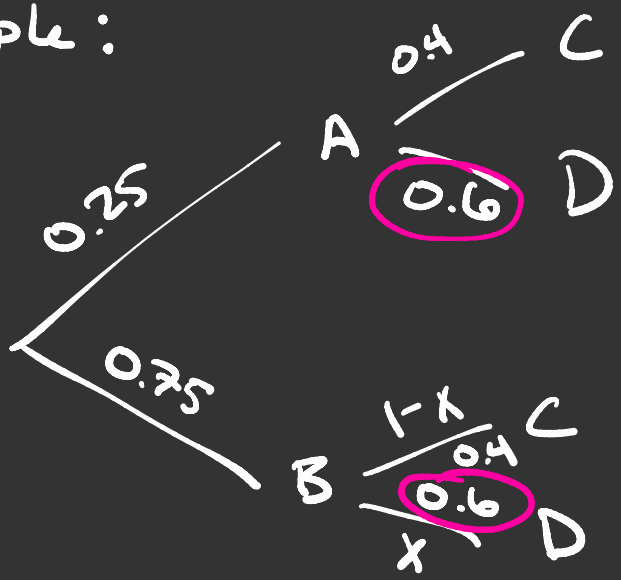
  $P(A) \cdot P(Def) = 0.5 \times 0.037 = 0.0185$

Not Independent

- $P(A) \stackrel{?}{=} P(A|Def)$

- $P(Def) \stackrel{?}{=} P(Def|A)$

Example:



What must x be so that A and D are independent?

$P(D|A) = P(D)$

$P(D|A) = 0.6$

$$P(D) = P(A \cap D) + P(B \cap D) = 0.25 \cdot 0.6 + 0.75 \cdot X$$
$$= 0.15 + 0.75X$$

$\rightsquigarrow 0.6 = 0.15 + 0.75X$

$0.45 = 0.75X \longrightarrow X = 0.6$