Independent Events

Recall: Events E and F are independent if

$$P(E|F) = P(E) \quad \text{or} \quad P(F|E) = P(F)$$

Question: How do we decide if two events are independent?

i) Compute $P(E|F)$, $P(E)$

ii) Compute $P(F|E)$, $P(F)$

iii) Multiplication Rule: If E, F are independent

Can check this equality instead

$$P(E \cap F) = P(F) \cdot P(E|F) = P(F) \cdot P(E)$$

Example: Select card from deck

E = card is red

F = card is 10

a) Are E, F independent?

$$P(E \cap F) = \frac{2}{52} = \frac{1}{26}$$

$$P(E) \cdot P(F) = \frac{26}{52} \cdot \frac{4}{52} = \frac{1}{2} \cdot \frac{1}{13} = \frac{1}{26}$$

equal → E, F independent

b) Are E, F mutually exclusive? No

Example: Jack, Jill working on problem

$P(\underbrace{\text{Jill solves problem}}_{A}) = \frac{4}{5}$ $P(\underbrace{\text{Jack solves problem}}_{B}) = \frac{1}{3}$

a) $P(\text{both solve}) = P(A \cap B) = P(A) \cdot P(B) = \frac{4}{5} \cdot \frac{1}{3} = \frac{4}{15}$

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b) $P(\text{neither solve}) = P(A' \cap B') = P(A') \cdot P(B') = \frac{1}{5} \cdot \frac{2}{3} = \frac{2}{15}$

c) $P(\text{exactly one solves}) = P(A \cap B') + P(A' \cap B)$

$= P(A) \cdot P(B') + P(A') \cdot P(B)$

$= \frac{4}{5} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{1}{3} = \frac{8}{15} + \frac{1}{15} = \frac{9}{15}$

Notice: For independent events, we can rewrite the Inclusion-Exclusion Principle as

$$P(E \cup F) = P(E) + P(F) - P(E) \cdot P(F)$$

Example: Compute P(at least one solves problem)

One way: $P(\geq 1 \text{ solves}) = P(A \cup B)$

$$= P(A) + P(B) - P(A) \cdot P(B)$$

$$= \frac{4}{5} + \frac{1}{3} - \frac{4}{5} \cdot \frac{1}{3}$$

$$= \frac{17}{15} - \frac{4}{15} = \frac{13}{15}$$

Second way: $P(\geq 1 \text{ solves}) = 1 - P(A' \cap B')$

$= 1 - \frac{2}{15} = \frac{13}{15}$

Third ways: $P(\geq 1 \text{ solves}) = P(\text{both solve}) + P(\text{exactly 1 solves})$

$= \frac{4}{15} + \frac{9}{15} = \frac{13}{15}$

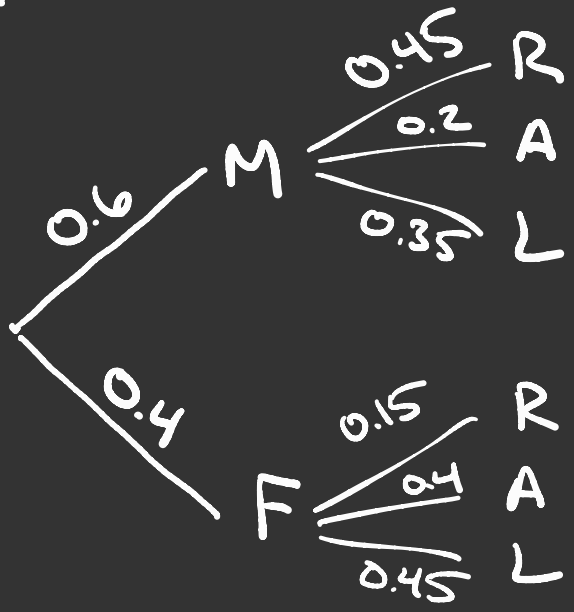
Example: City council voting on project to fund:

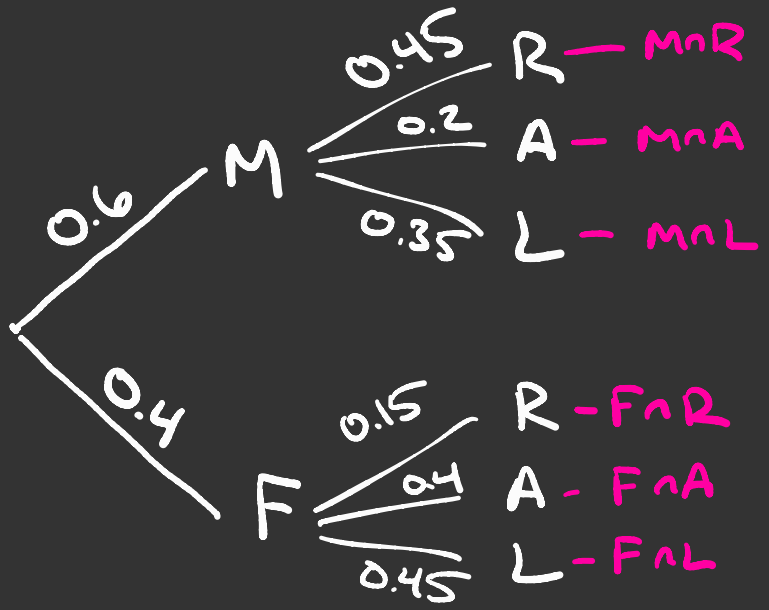
- Rec Center
- Arts center
- Library

Council has 120 men, 80 women

Men: 45% rec
20% arts
35% library

Women: 15% rec
40% arts
45% library





$P(M) = 0.6$

$P(F) = 0.4$

$P(R|M) = 0.45$

$P(M \cap R) = 0.6 \cdot 0.45 = 0.27$

$P(A) = P(M \cap A) + P(F \cap A)$

$= 0.6 \cdot 0.2 + 0.4 \cdot 0.4$

$= 0.12 + 0.16 = 0.28$

$P(F \text{ and } (A \text{ or } L)) = P(F \text{ and } A) + P(F \text{ and } L)$

$= 0.4 \cdot 0.4 + 0.4 \cdot 0.45$

$= 0.16 + 0.18 = \underline{0.34}$