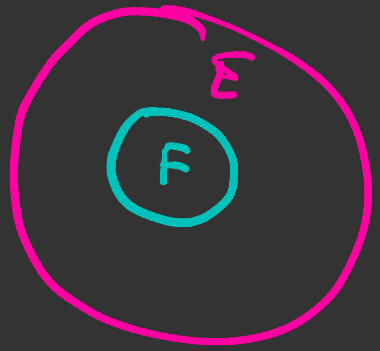
Warm-Up: Suppose F subset of E. What is $P(E|F)$ and $P(F|E)$?

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$



$$P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{P(F)}{P(E)}$$

Example: Rolling a die. E = even, $F = 2$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/6} = 1$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{1/6}{1/2} = \frac{1}{3}$$

Multiplication Rule:

$$P(E \cap F) = P(F) \cdot P(E|F)$$

Example: 12 light bulbs in box, 3 defective. 3 bulbs selected at random. What is the prob that all 3 are defective?

$$P(\text{first def and second def and third def})$$

$$= P(\text{first def}) \cdot P(\text{second def} \mid \text{first def}) \cdot P(\text{third def} \mid \text{first and second def})$$

$$= \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \frac{6}{1320}$$

Example:

a) Two players flip coins. What is the prob they both get heads?

$$P(\text{first heads and second heads}) = P(\text{first heads}) \cdot P(\text{second heads} \mid \text{first heads}) = \frac{1}{2} \cdot \frac{1}{2}$$

b) Two players dealt one card. What is the prob they both get aces?

$$P(\text{first ace and second ace}) = P(\text{first ace}) \cdot P(\text{second ace} \mid \text{first ace}) = \frac{4}{52} \cdot \frac{3}{51}$$

(a) $\rightsquigarrow$ first event did not affect the second

Def: Events E and F are independent if

$$P(E|F) = P(E) \quad \text{or} \quad P(F|E) = P(F)$$

Only need to check one

Otherwise, dependent

Example: 3 toys (red, green, blue). 2 children each pick one.

E = first kid picks red toy

F = Second kid pick green toy.

a) With replacement

$$P(F|E) = \frac{1}{3} \qquad P(F) = \frac{3}{9} = \frac{1}{3}$$

Independent

b) Without replacement

$$P(F|E) = \frac{1}{2} \qquad P(F) = \frac{2}{6} = \frac{1}{3}$$

$\neq$ Dependent

Warning: Don't confuse independent w/ mutually exclusive.

E & F are mutually exclusive $\rightsquigarrow$ $P(E|F) = 0$

Mutually exclusive events w/ nonzero prob must be dependent

Example: 200 students in school

50 freshmen 20 students in Math club

5 student in Math club are freshmen

$P(\text{freshman}) = \frac{1}{4}$ $P(\text{freshman} \mid \text{in club}) = \frac{1}{4}$

$P(\text{in club}) = \frac{1}{10}$ $P(\text{in club} \mid \text{freshman}) = \frac{1}{10}$

E = freshmen
F = in Math club

} Independent events.

Example: Pick random American person

E = University of Michigan fan

F = stupid jerk face

Not mutually exclusive.

$P(F|E) = 1$ $\quad$ $P(F) < 1$