Example: 200 students. 40 take English, 50 take Math, 10 take both. What is the probability that a student randomly drawn from the math class is in English?

Total choices: 50 students

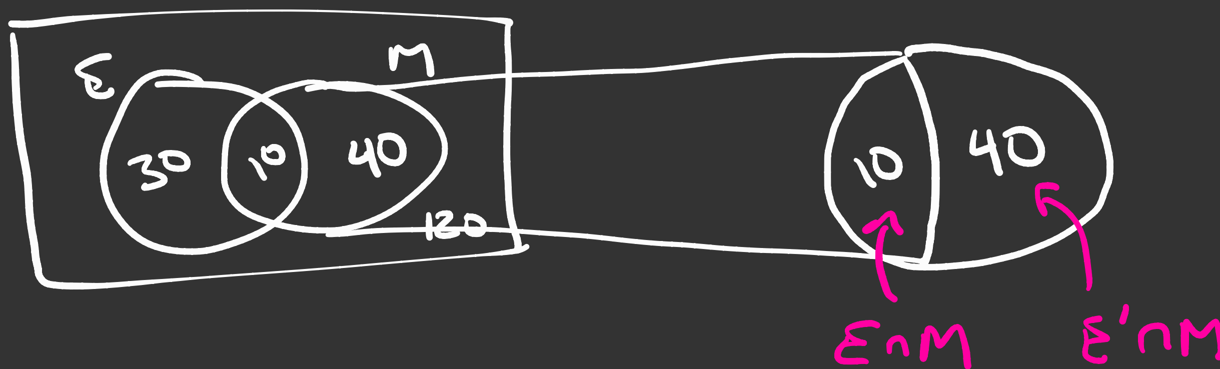
\# of those in English: 10 students

$\rightarrow \frac{10}{50} = 0.2$

$P(E|M)$ = "the probability of E <u>given</u> M"

$$= \frac{n(E \cap M)}{n(M)}$$

Intuition: We are restricting the sample space to M



Def: E and F events in sample space S, with $P(F) \neq 0$

The conditional probability of E given F is

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

In equally likely events, we have $P(E|F) = \dfrac{n(E \cap F)}{n(F)}$

Example:

a) Student selected from English class. What is the prob that they're in Math?

$$P(M|E) = \frac{n(M \cap E)}{n(E)} = \frac{10}{40} = 0.25$$

b) Student selected from English class. What is the prob that they're <u>not</u> in Math?

$$P(M'|E) = \frac{n(M' \cap E)}{n(E)} = \frac{30}{40} = 0.75$$

Another way: $P(M'|E) = 1 - P(M|E) = 1 - 0.25$

c) Student selected from English class. What is the prob that they're in English?

$$P(E|E) = \frac{n(E \cap E)}{n(E)} = \frac{40}{40} = 1$$

In general, $P(F|F) = 1$ for any event F (assuming $P(F) \neq 0$)

Properties of $P(E|F)$:

i) $P(E'|F) = 1 - P(E|F)$

ii) $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$

Multiplication Rule

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \longrightarrow P(E \cap F) = P(F) \cdot P(E|F)$$

New way to compute prob of intersections

Example: Two cards from a deck, no replacement. What is the prob that we draw ace first, then king?

$$P(\text{ace } 1^{st} \cap \text{king } 2^{nd}) = P(\text{ace } 1^{st}) \cdot P(\text{king } 2^{nd} \mid \text{ace } 1^{st})$$

F = ace 1^{st}

E = king second

$$= \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$$