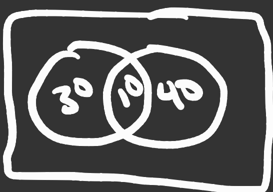
Goal for today: Find probabilities of unions, intersections, and complements.

Example: 200 students. 40 take English, 50 take math, 10 take both. E = English M = Math



$$P(E) = \boxed{\frac{40}{200}} = 0.2 \qquad P(M) = \frac{50}{200} = 0.25$$

$$P(\text{not } E) = P(E') = \frac{200-40}{200} = 0.8$$

$$= \frac{200}{200} - \frac{40}{200} = 1 - P(E)$$

$$P(E \text{ or } M) = P(E \cup M) = \frac{n(E \cup M)}{n(S)}$$

$$= \frac{n(E) + n(M) - n(E \cap M)}{200}$$

$$= \frac{40 + 50 - 10}{200}$$

$$= \frac{40}{200} + \frac{50}{200} - \frac{10}{200}$$

$$= P(E) + P(M) - P(E \cap M)$$

Theorem: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ for any events $E + F$

Defintion: Events E and F are <u>mutually exclusive</u> if they have no outcomes in common (i.e. $E \cap F = \emptyset$)

Examples:

- Flipping a coin, E = Heads F = Tails

  mutually exclusive

- Flip 2 coins E = at least 1 Heads
  F = at least 1 Tails

  Not mut exclusive

- Rolling a pair of dice, E = sum is 7 F = sum is 9
  <u>are</u> mutually exclusive

  E = sum is 7 F = at least one die shows a 3

  not mut exclusive

If E and F mutually exclusive events then

$$P(E \cup F) = P(E) + P(F)$$

Examples: • Flipping a coin $\quad E = \text{Heads} \quad F = \text{tails}$

$$P(E \cup F) = P(E) + P(F)$$
$$= \frac{1}{2} + \frac{1}{2} = 1$$

• Rolling pair of dice $\quad E = \text{sum is } 7 \quad F = \text{sum is } 9$

$$P(E \cup F) = P(E) + P(F) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$$

• Rolling two dice E = sum is 7 F = at least one die shows a 3

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= \frac{6}{36} + \frac{11}{36} - \frac{2}{36} = \frac{15}{36} = \frac{5}{12}$$

Example: Two people selected from a group of 7 men and 5 women

a) $P(\text{both men or both women}) = P(\text{both men}) + P(\text{both women})$
$$= \frac{C(7,2)}{C(12,2)} + \frac{C(5,2)}{C(12,2)}$$

b) $P(\text{at least one is a man}) = 1 - P(\text{none are men}) = 1 - P(\text{both women})$
$$= 1 - \frac{C(5,2)}{C(12,2)}$$

Question: How many people need to be in a group to guarantee that 2 people share a birthdate?

366 people

Question: How many to have a 50% chance that 2 people share a birthday?

Start with a group of 5 people

$$P(\geq 2 \text{ people share a birthday}) = 1 - P(\text{everyone has different bday})$$

$$= 1 - \frac{P(365,5)}{365^5} = 0.027$$

For n people, probability that a bday is shared is

$$1 - \frac{P(365, n)}{365^n}$$