

Equally Likely Events: E an event, sample S

$$P(E) = \frac{n(E)}{n(S)}$$

Example: Roll two dice (one white/one blue). What is the probability that the sum is 8?

Sample space: $\{1, 2, 3, \dots, 11, 12\}$

← Not equally likely events

Sample space: $(1, 1) \quad (1, 2) \quad \dots \quad (1, 6)$

$(5, 3) \quad (4, 4) \quad (3, 5) \quad (2, 6)$

$(6, 1) \quad (6, 2) \quad \dots \quad (6, 6)$

} 36 elements

$$P(\text{sum is } 8) = \frac{5}{36}$$

Example: Coin flipped 3 times. What is the probability of at least two heads?

Sample space: $\{ \underline{HHH}, \underline{HHT}, \underline{HTH}, \underline{THH}, \overbrace{TTH, THT, HTT}^{TTT} \}$

$$P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2} = \frac{C(3,2) + C(3,3)}{8} = \frac{3+1}{8} = \frac{1}{2}$$

Coin flipped 11 times. What is the prob of getting at least 6 heads?

$$P(\text{at least 6 heads}) = \frac{C(11,6) + C(11,7) + C(11,8) + \dots + C(11,11)}{2^{11}}$$

Example: Nine cards labeled 1-9, randomly choose 3
different cards
in order to make 3-digit number. What is prob this
number is 500 or bigger?

Sample space has $P(9,3) = 9 \cdot 8 \cdot 7$ elements

5 · 8 · 7 ways to get ≥ 500

$$P(\geq 500) = \frac{5 \cdot 8 \cdot 7}{9 \cdot 8 \cdot 7} = \frac{5}{9}$$

Example: 15 cards labeled 1-15. 3 randomly and placed in row (in order). What is the probability that

a) all three are odd? Sample space: $15 \cdot 14 \cdot 13$ elements

$$P(\text{all odd}) = \frac{8 \cdot 7 \cdot 6}{15 \cdot 14 \cdot 13} = 0.123$$

b) first two odd, third even?

$$P(\downarrow) = \frac{8 \cdot 7 \cdot 7}{15 \cdot 14 \cdot 13} = 0.144$$

c) all 3 are larger than 10?

$$P(\downarrow) = \frac{5 \cdot 4 \cdot 3}{15 \cdot 14 \cdot 13} = 0.022$$

1	2	3	Switching wins
Goat	Goat	Car	No
Goat	Car	Goat	Yes
Car	Goat	Goat	Yes