Sample space: Set of all possible outcomes

Simple outcomes: Elements of sample space

Properties of sample spaces:

a) Contain element for every outcome

({fresh, soph, jun} would <u>not</u> be a good sample space)

b) Each element corresponds to <u>exactly one</u> element of space. ({blue eyes, brown eyes, brown hair, blonde hair, ...} is no good.)

Event: subset of sample space

Simple event: Event contain <u>one</u> simple outcome

Note: $\emptyset$ and entire sample space are both events.

Say an event <u>occurs</u> if trial yields an outcome belonging to that event.

Example: Rolling 6-sided die

$S = \{1, 2, 3, 4, 5, 6\}$

E = event of rolling even number $= \{2, 4, 6\}$

Do a trial $\rightsquigarrow$ roll a 4 $\rightsquigarrow$ E <u>occurs</u>.

Properties of probability: Sample space $S = \{e_1, e_2, \dots, e_n\}$

simple outcomes

1) Each e_i is assigned a probability $P(e_i)$

2) Probability of event = sum of probabilities of simple outcomes

$$P(\{e_1, e_2, e_3\}) = P(e_1) + P(e_2) + P(e_3)$$

3) $0 \leq P(e_i) \leq 1$ and $0 \leq P(E) \leq 1$

any event

4) $P(S) = 1$ $\qquad P(\emptyset) = 0$

Example: $S = \{A, B, C, D\}$

$P(A) = 0.35 \quad P(B) = 0.15 \quad P(C) = 0.22 \quad P(D) = 0.28$

$P(\{A, B\}) = P(A) + P(B) = 0.35 + 0.15 = 0.5$

$P(\{B, C, D\}) = P(B) + P(C) + P(D) = 0.65$

$P(\{A, B, C, D\}) = 1$

Example: Cooler at picnic

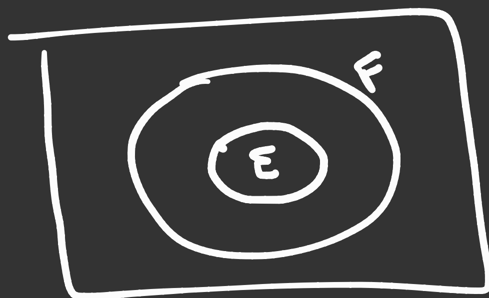
	Pepsi	Coke	Dr Peppers
Reg	0.05	0.15	0.23
Diet	0.10	0.17	0.30

$$P\left(\begin{matrix}\text{Any kind}\\ \text{of coke}\end{matrix}\right) = 0.15 + 0.17 = 0.32$$

$$P(\text{Diet}) = 0.10 + 0.17 + 0.30 = 0.57$$

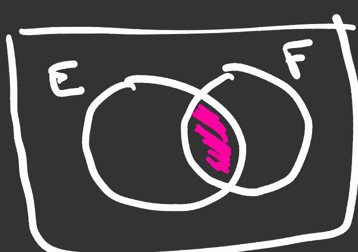
$$P(\text{not Dr Pepper}) = 0.05 + 0.10 + 0.15 + 0.17$$
$$= 1 - (0.23 - 0.30) = 0.47$$

Probabilities of Unions/Intersections/Subsets



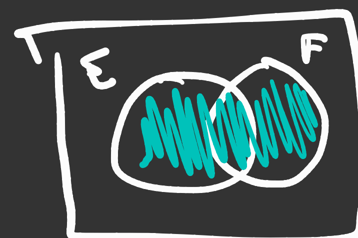
$P(E) \leq P(F)$

$E \subseteq F$



$P(E \text{ and } F)$
$= P(E \cap F) \leq P(E)$

$E \cap F$



$P(E \text{ or } F)$
$= P(E \cup F) \geq P(E)$

$E \cup F$

Equally likely events

Example: Choose a number at random from $\{1, 2, 3, \ldots, 9\}$

$E = \text{even} = \{2, 4, 6, 8\}$

$$P(E) = \frac{4}{9} \quad \begin{array}{l} \leftarrow \text{\# of elements in } E = n(E) \\ \leftarrow \text{\# of elements in } S = n(S) \end{array}$$

Theorem: If S is a sample space, E an event, and all simple outcomes are equally likely, then

$$P(E) = \frac{n(E)}{n(S)} \qquad P(\text{not } E) = 1 - \frac{n(E)}{n(S)}$$

$$= \frac{n(S) - n(E)}{n(S)}$$

Example: Bookstore has 40 books, 17 mystery and 23 romance. What is the prob a randomly chosen book is mystery?

$$P(\text{mystery}) = \frac{n(\text{mystery})}{n(\text{total books})} = \frac{17}{40}$$

Example: Playing lottery. Sample space = $\{$win jackpot, don't win jackpot$\}$

$$P(\text{win}) = \frac{n(\text{win})}{n(S)} \neq \frac{1}{2}.$$

Events are <u>not</u> equally likely.