

## Practice Exam 1

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
| 2.  | (a) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (e) |
| 4.  | (a) | (b) | (c) | (d) | (e) |
| 5.  | (a) | (b) | (c) | (d) | (e) |
| 6.  | (a) | (b) | (c) | (d) | (e) |
| 7.  | (a) | (b) | (c) | (d) | (e) |
| 8.  | (a) | (b) | (c) | (d) | (e) |
| 9.  | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

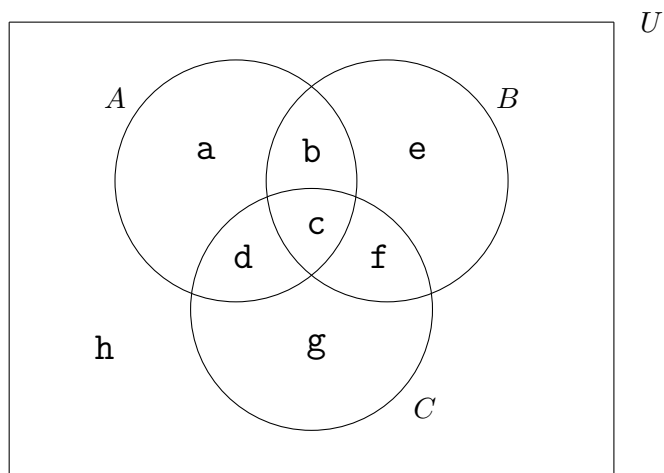
14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) In the following Venn diagram, which of the following is equal to  $(A \cap B) \cup C'$ ? (Note the “prime” over the  $C$ .)



- (a)  $\{a, b, e, h\}$                       (b)  $\{b\}$                                       (c)  $\{a, b, c, e, h\}$
- (d)  $\{a, b, e\}$                               (e)  $\{a, b, c, e, g\}$

2. (5 pts.) A small technical college has 200 students. They have 100 math majors, 80 physics majors, and 20 double majors in both math and physics. How many students are majoring in physics but not math? (A Venn diagram might be helpful.)

- (a) 60                      (b) 40                      (c) 80                      (d) 100                      (e) none

3. (5 pts.) Bob owns four pairs of pants and six shirts. Before a trip, he wants to choose two pairs of pants and three shirts to pack in his suitcase. In how many ways can he do this?

(a)  $C(4, 2) + C(6, 3)$

(b)  $P(4, 2) + P(6, 3)$

(c)  $P(4, 2) \cdot P(6, 3)$

(d)  $4! \cdot 6!$

(e)  $C(4, 2) \cdot C(6, 3)$

4. (5 pts.) A standard deck consists of 52 cards, with an A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K of each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. In how many ways can a person be given two A's together with three other cards that are not A's (for a total of five cards)? [Hint: all that matters is which cards the person winds up with, not the order in which she receives them.]

(a)  $C(4, 2) + C(48, 3)$

(b)  $P(4, 2) \cdot P(48, 3)$

(c)  $P(4, 2) + P(48, 3)$

(d)  $C(4, 2) \cdot C(48, 3)$

(e)  $4^2 \cdot 48^3$

5. (5 pts.) A license plate in a certain state features 3 letters (repetition NOT allowed) followed by 3 digits (repetition IS allowed). How many different license plates are possible? [Hint: there are 26 letters and 10 digits. Obviously ABC-112 is different from CBA-211.]

- (a)  $P(26, 3) \cdot 10^3$                       (b)  $P(26, 3) \cdot P(10, 3)$                       (c)  $C(26, 3) \cdot C(10, 3)$   
 (d)  $C(26, 3) \cdot 10^3$                       (e)  $26^3 \cdot 10^3$

6. (5 pts.) Suppose I flip a coin seven times and record the sequence of heads and tails (e.g. HHTHTTT). How many such sequences are there with exactly three heads and four tails?

- (a) 1225                      (b) 35                      (c) 12                      (d) 7                      (e) 128

# ways to fill 7 blanks w/ 3 H's =  $C(7, 3) = C(7, 4)$   

$$\frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$$

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards from this deck.

How many Poker hands have three spades and two clubs?

- (a)  $C(13, 5)$
- (b)  $C(13, 3) + C(13, 2)$
- (c)  $P(13, 3) \cdot P(13, 2)$
- (d)  $C(52, 3) + C(52, 2)$
- (e)  $C(13, 3) \cdot C(13, 2)$

8. (5 pts.) A club has 40 members. Because the governor is coming to give a speech to them, they break into four committees: a 10-member welcoming committee, a 10-member decoration committee, a 10-member food committee and a 10-member protest committee to make protest banners and signs. In how many ways can they divide themselves into these committees, taking into account that being on one committee is different from being on another committee?

- (a)  $\frac{1}{4!} \cdot \frac{40!}{10! \cdot 10! \cdot 10! \cdot 10!}$
- (b)  $\frac{40!}{10! \cdot 10! \cdot 10! \cdot 10!}$
- (c)  $\frac{40!}{30! \cdot 20! \cdot 10! \cdot 0!}$
- (d)  $\frac{1}{4!} \cdot \frac{40!}{30! \cdot 20! \cdot 10! \cdot 0!}$
- (e)  $\frac{1}{4!} \cdot \frac{40!}{30! \cdot 10!}$

$$\frac{1}{4!} \cdot \frac{40!}{10! \cdot 10! \cdot 10! \cdot 10!}$$

ordered partitions

Unordered partitions

Want ordered partitions

9. (5 pts.) Suppose I roll a normal 6-sided die seven times and record the resulting sequence of numbers. How many sequences are there that contain **exactly** three 4's? (Don't forget that there are **seven** rolls, and that there are five other numbers that can come up besides a 4, and numbers may be repeated.)

- (a)  $C(7, 3)$                       (b)  $P(7, 3)$                       (c)  $C(7, 3) \cdot 5^4$   
 (d)  $6^3$                               (e)  $C(7, 3) \cdot 6^4$

10. (5 pts.) A club has 10 members. They are planning a big party, and they need to choose an organizing committee. This committee has to have **at least** two members (it could be more than two, but it can't consist of no one, and it can't consist of just one member). In how many ways can they choose this committee?

- (a)  $2^{10} = 1024$                       (b)  $2^{10} - 10 = 1014$                       (c)  $C(10, 2) - 1 - 10 = 34$   
 (d)  $2^{10} - 1 - 10 = 1013$                       (e)  $C(10, 2) = 45$

$$\begin{aligned} \# \text{ committees } &= \# \text{ of total } - \# \text{ w/ 1 } - \# \text{ w/ 0} \\ \text{w/ } \geq 2 \text{ people} & \text{ committees } \quad \text{person} \quad \text{people} \\ &= 2^{10} - C(10, 1) - C(10, 0) \\ &= 1024 - 10 - 1 \\ &= 1013 \end{aligned}$$

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) A club has 9 members: 4 men and 5 women. For each of the following parts of this problem, give a numerical answer (e.g. if the answer should be  $C(4, 2)$ , write 6.) These questions should be assumed to be independent of each other.

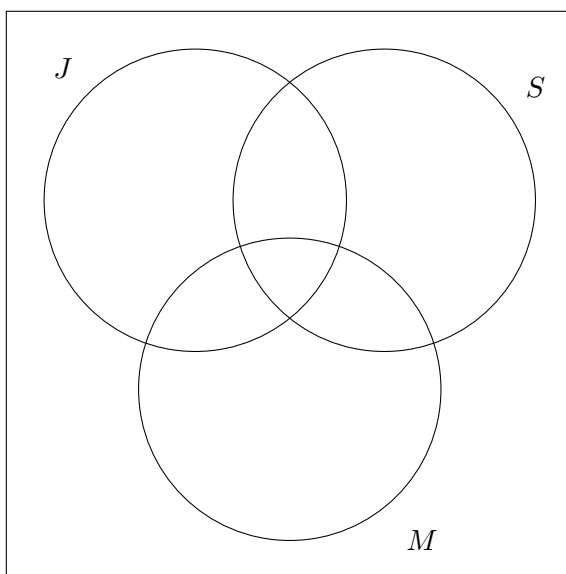
(a) In how many ways can they choose an executive committee of four people, if gender is irrelevant?

(b) In how many ways can they choose an executive committee consisting of 2 men and 2 women?

**12.** (10 pts.) A certain college has 1100 students. We have the following information about the clubs that they belong to.

- 400 belong to the juggling club.
- 600 belong to the science club.
- 500 belong to the Mock Trial club
- 110 belong to both the juggling club and the science club.
- 250 belong to both the science club and the Mock Trial club.
- 150 belong to both the juggling club and the Mock Trial club.
- 60 belong to all three clubs.

Fill in **all** regions of the following Venn diagram, where  $J$  represents the juggling club,  $S$  represents the science club and  $M$  represents the Mock Trial club.





13. (10 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 10 times and write down the sequence of heads and tails that shows up.

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

(a) How many different sequences of heads and tails are possible?

$$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} = 2^{10} = 1024$$

(b) In how many ways can I get a total of 5 heads with the first and last toss being heads?

$\underline{H}$  — — — — —  $\underline{H}$   
 {  
 3 H's and 5 T's  
 $C(8, 3)$  or  $C(8, 5)$

**14.** (10 pts.)

A bag contains 9 colored marbles, of which 5 are red and 4 are blue. (Assume that the marbles are distinguishable from each other.) I plan to pick 3 marbles from the bag.

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

(a) What is the total number of ways 3 marbles can be selected (ignoring color)?

(b) If I pick 3 marbles, in how many ways can I get all red marbles or all blue marbles?

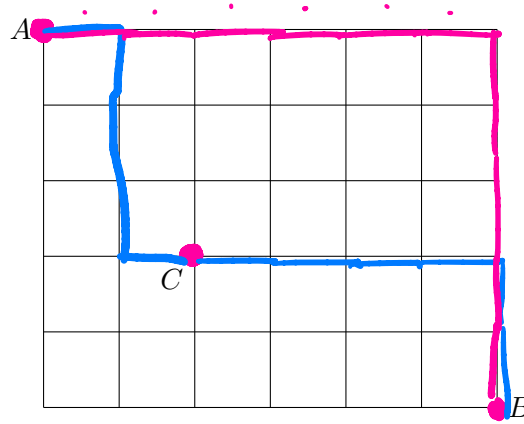
15. (10 pts.) In this problem, be sure to show all your work and be sure to plainly mark your answers.

The following is a street map of part of a city. Emily lives at the northwest corner (marked  $A$ ) and wants to get to the library at the southeast corner (marked  $B$ ). She only travels east and south (i.e. to the right or down), following the roads. She happens to owe \$50 to Claire, who lives at the corner marked  $C$ .

R D D D R

— — — — —

$C(5,3) =$  # ways to fill in the D's



# of total paths from  $A \rightarrow B$

$\equiv$

# of paths from  $A \rightarrow B$  starting with R

$\rightarrow +$

# paths from  $A \rightarrow B$  starting with D

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

(a) If she decides to be conscientious and pay Claire her \$50, how many different routes are there from  $A$  to  $B$  that DO pass through  $C$ ?

$$\# \text{ paths from } A \rightarrow B \text{ through } C = \# \text{ paths from } A \rightarrow C \times \# \text{ paths from } C \rightarrow B = C(5,3) \times C(6,4)$$

(b) If she decides she wants to hang on to her money for now, how many different routes are there from  $A$  to  $B$  that do NOT pass through  $C$ ?

$$\begin{aligned} \# \text{ paths from } A \rightarrow B \text{ not going through } C &= \# \text{ of total paths from } A \rightarrow B - \# \text{ of paths from } A \rightarrow B \text{ going through } C \\ &= C(11,5) - (C(5,3) \times C(6,4)) \end{aligned}$$

---

11 blanks. Fill in D's first, then R's

$C(11,5) \times 1$

## Practice Exam 1

February 6, 2020

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.**

The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (●) | (d) | (e) |
| 2.  | (●) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (●) |
| 4.  | (a) | (b) | (c) | (●) | (e) |
| 5.  | (●) | (b) | (c) | (d) | (e) |
| 6.  | (a) | (●) | (c) | (d) | (e) |
| 7.  | (a) | (b) | (c) | (d) | (●) |
| 8.  | (a) | (●) | (c) | (d) | (e) |
| 9.  | (a) | (b) | (●) | (d) | (e) |
| 10. | (a) | (b) | (c) | (●) | (e) |

MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_

## Practice Exam 1

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
| 2.  | (a) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (e) |
| 4.  | (a) | (b) | (c) | (d) | (e) |
| 5.  | (a) | (b) | (c) | (d) | (e) |
| 6.  | (a) | (b) | (c) | (d) | (e) |
| 7.  | (a) | (b) | (c) | (d) | (e) |
| 8.  | (a) | (b) | (c) | (d) | (e) |
| 9.  | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

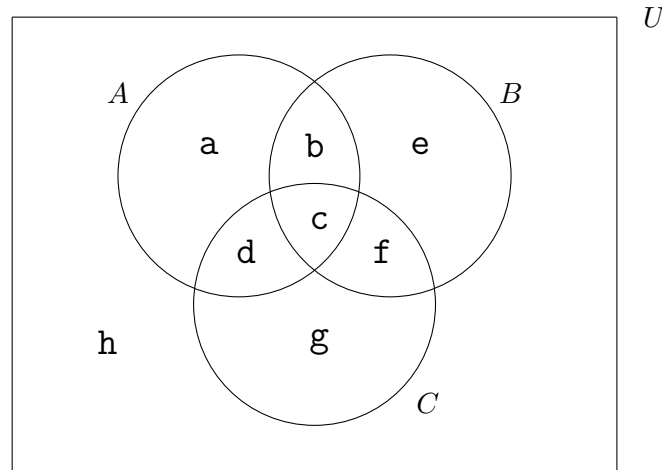
14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) In the following Venn diagram, which of the following is equal to  $A \cap (B \cup C)'$ ? (Note the “prime” over the  $C$ .)



- (a)  $\{b, c, d\}$                       (b)  $\{a, h\}$                       (c)  $\{a, b, c, d\}$
- (d)  $\{e, f, g\}$                       (e)  $\{a\}$

2. (5 pts.) Emily has a nice collection of poetry anthology books. Of these, 50 include poems by Robert Frost (among other authors), 40 include poems by Pablo Neruda (among other authors), 30 include poems by both Frost and Neruda (among other authors), and 20 don't have poems by either Frost or Neruda. How many poetry books are in her collection?

- (a) 110                      (b) 80                      (c) 60                      (d) 120                      (e) 70

3. (5 pts.) Claire is running for president, and for her next campaigning trip she plans to visit four states. For her first two stops, her choices are Alabama, Alaska, Arizona and Arkansas (there are four of them). For her third and fourth stops her choices are Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri and Montana (there are eight of them). How many different 4-state campaigning trips are possible? (Order is important in this problem – Alaska then Alabama is different from Alabama then Alaska.)

- (a)  $P(4, 1) + P(4, 1) + P(8, 1) + P(8, 1)$
- (b)  $[P(4, 1) \cdot P(4, 1)] + [P(8, 1) \cdot P(8, 1)]$
- (c)  $4! \cdot 8!$
- (d)  $P(4, 2) \cdot P(8, 2)$
- (e)  $C(4, 2) \cdot C(8, 2)$

4. (5 pts.) A standard deck consists of 52 cards, with an A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K of each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. In how many ways can a person be given a 2, a 3, a 4, a 5 and a 6?

- (a)  $\frac{C(4, 1)}{5!}$
- (b)  $P(4, 3)^5$
- (c)  $\frac{P(4, 3)}{5!}$
- (d)  $C(4, 1)^5$
- (e)  $C(4, 1) + C(4, 1) + C(4, 1) + C(4, 1) + C(4, 1)$

5. (5 pts.) When you get an account at a certain website, you have to choose a password consisting of five letters (with NO repetition) followed by two digits (repetitions allowed). How many different passwords are possible? [Hint: there are 26 letters and 10 digits. Obviously ABCDE-33 is different from CEBDA-33.]

(a)  $P(26, 5) \cdot 10^2$

(b)  $P(26, 5) \cdot P(10, 2)$

(c)  $C(26, 5) \cdot C(10, 2)$

(d)  $C(26, 5) \cdot 10^2$

(e)  $26^5 \cdot 10^2$

6. (5 pts.) A die has six sides, labelled 1 to 6. Suppose I roll the die three times and record the sequence of results (e.g. 4-1-5). How many such sequences don't have **any** 3's or 4's?

(a) 152

(b) 200

(c) 64

(d) 204

(e) 120



7. (5 pts.) A club has 9 members, and they have to choose an organizing committee of three people for an upcoming event. In how many ways can they do this?

- (a) 24                      (b) 84                      (c) 729                      (d) 27                      (e) 504

8. (5 pts.) Six couples decide to go to the movies and sit together in the front row. In how many ways can these 12 people seat themselves if couples must sit together? [Hint: looking left to right, Bob-Sally is different from Sally-Bob.]

- (a) 46,080                      (b) 720                      (c) 10,800  
(d) 21,600                      (e) 8,640

9. (5 pts.) A dinner at Luigi's pizza consists of a pizza and a salad. They offer six different toppings, of which a customer can choose as many as they like as long as they choose at least one. They offer three different salads, of which the customer has to choose exactly one. How many dinners are possible?

- (a) 66                      (b) 67                      (c) 192                      (d) 18                      (e) 189

10. (5 pts.) A box of candy contains 30 different chocolates. Mom has three indistinguishable paper bags and decides to put 10 candies in each bag. The order of the bags makes no difference because they're indistinguishable. In how many ways can she divide the chocolates into the three bags?

- (a)  $\frac{1}{6} \cdot \frac{30!}{10! \cdot 10! \cdot 20!}$                       (b)  $\frac{30!}{10! \cdot 20!}$                       (c)  $\frac{1}{6} \cdot \frac{30!}{10! \cdot 10! \cdot 10!}$   
 (d)  $\frac{30!}{10! \cdot 10! \cdot 10!}$                       (e)  $\frac{1}{720} \cdot \frac{30!}{10! \cdot 10! \cdot 10!}$

$\frac{1}{3!} \cdot \frac{30!}{10! \cdot 10! \cdot 10!}$                       Want unordered partitions

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit!

Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) A club has 10 members: 4 men and 6 women. For each of the following parts of this problem, give a numerical answer (e.g. if the answer should be  $C(4, 2)$ , write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose an executive committee of four people, if gender is irrelevant?

**Answer to (a):**

← Remember this has to be a number!

(b) In how many ways can they choose an executive committee consisting of 2 men and 2 women?

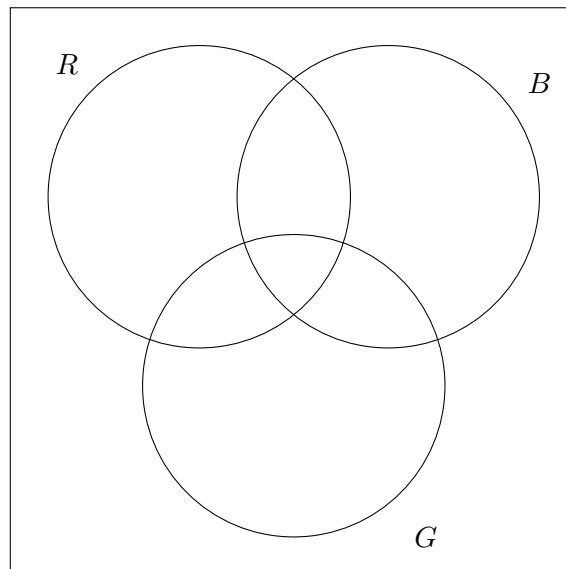
**Answer to (b):**

← Remember this has to be a number!

**12.** (10 pts.) A horror movie has a cast of 250 clowns. Some have red noses ( $R$ ), some have blue shirts ( $B$ ) and some have green pants ( $G$ ).

- 110 have red noses.
- 130 have blue shirts.
- 100 have green pants.
- 30 have both red noses and blue shirts.
- 70 have both blue shirts and green pants.
- 40 have both red noses and green pants.
- 10 have all three: red noses, blue shirts and green pants.

Fill in **all** regions of the following Venn diagram.



**13.** (10 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 12 times and write down the sequence of heads and tails that shows up.

**Note:** In the following two parts, it is **not** necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

(a) How many different sequences of heads and tails are possible?

**Answer to (a):**

(b) In how many ways can I get a total of 4 heads with the first being heads and the last being tails? [Hint: the first one is heads, so you just need to have three more heads.]

**Answer to (b):**

**14.** (10 pts.)

A bag contains 12 colored marbles, of which 5 are red, 4 are white and 3 are blue. (Assume that the marbles are distinguishable from each other. In this problem, the order that you pick the marbles does not matter.) I plan to pick 3 marbles from the bag.

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

- (a) In how many ways can I pick the three marbles so that they are all the same color?

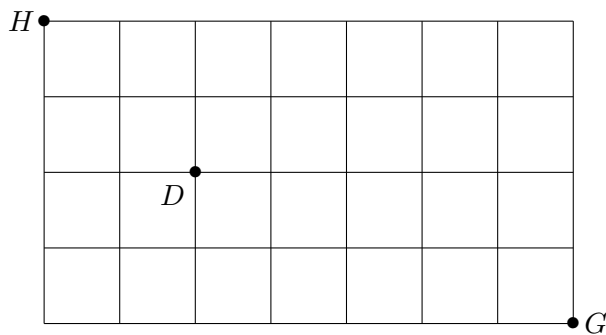
**Answer to (a):**

- (b) In how many ways can I pick the three marbles so that there are strictly more red marbles than either of the other colors? [Hint: as a first step, figure out all the ways that more of the three can be red than either white or blue. A tie is no good, i.e. 1 red, 1 white and 1 blue does not contain strictly more red than white or blue.]

**Answer to (b):**

15. (10 pts.) In this problem, be sure to show all your work and be sure to use the answer box.

The following is a street map of part of the city where Harry Potter lives with his uncle and aunt. Harry lives at the northwest corner (marked  $H$ ) and wants to get to the grocery store at the southeast corner (marked  $G$ ). He only travels east and south (i.e. to the right or down), following the roads. Unfortunately, there is a dementor at the corner marked  $D$ .



**Note:** In the following two parts, you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

- (a) If he doesn't care whether he meets the dementor or not, how many paths are possible for him to get from  $H$  to  $G$ ?

**Answer to (a):**

- (b) If he would rather not meet the dementor, how many paths are possible for him to get from  $H$  to  $G$  that do **not** pass through  $D$ ?

**Answer to (b):**

## Exam 1

February 6, 2020

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (●) |
| 2.  | (a) | (●) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (●) | (e) |
| 4.  | (a) | (b) | (c) | (●) | (e) |
| 5.  | (●) | (b) | (c) | (d) | (e) |
| 6.  | (a) | (b) | (●) | (d) | (e) |
| 7.  | (a) | (●) | (c) | (d) | (e) |
| 8.  | (●) | (b) | (c) | (d) | (e) |
| 9.  | (a) | (b) | (c) | (d) | (●) |
| 10. | (a) | (b) | (●) | (d) | (e) |

MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_