

**Math 10120 Finite Math.**  
**Practice Final Exam 2**  
**May 8, 2019**

Name: \_\_\_\_\_

- Be sure that you have all 20 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

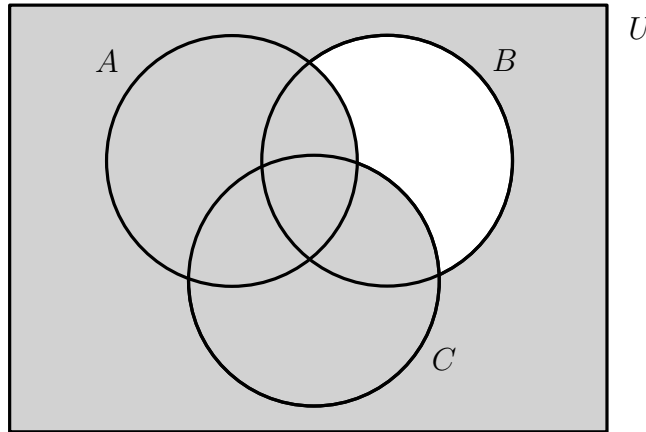
PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                                    |
|-------------------------|------------------------------------|
| 1. (a) (b) (c) (d) (e)  | 17. (a) (b) (c) (d) (e)            |
| 2. (a) (b) (c) (d) (e)  | 18. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 3. (a) (b) (c) (d) (e)  | 19. (a) (b) (c) (d) (e)            |
| 4. (a) (b) (c) (d) (e)  | 20. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 5. (a) (b) (c) (d) (e)  | 21. (a) (b) (c) (d) (e)            |
| 6. (a) (b) (c) (d) (e)  | 22. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 7. (a) (b) (c) (d) (e)  | 23. (a) (b) (c) (d) (e)            |
| 8. (a) (b) (c) (d) (e)  | 24. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 9. (a) (b) (c) (d) (e)  | 25. (a) (b) (c) (d) (e)            |
| 10. (a) (b) (c) (d) (e) | 26. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 11. (a) (b) (c) (d) (e) | <del>27. (a) (b) (c) (d) (e)</del> |
| 12. (a) (b) (c) (d) (e) | <del>28. (a) (b) (c) (d) (e)</del> |
| .....                   |                                    |
| 13. (a) (b) (c) (d) (e) | <del>29. (a) (b) (c) (d) (e)</del> |
| 14. (a) (b) (c) (d) (e) | <del>30. (a) (b) (c) (d) (e)</del> |
| .....                   |                                    |
| 15. (a) (b) (c) (d) (e) |                                    |
| 16. (a) (b) (c) (d) (e) |                                    |

Name: \_\_\_\_\_

Multiple Choice

1. (5 pts.) Which of the following corresponds to the shaded area in the Venn diagram below?  
(Recall that the notation  $( )'$  refers to the complement of the set  $( )$ .)



- (a)  $B \cap (A \cup C)'$       ~~(b)~~  $B' \cup (A \cup C)$       (c)  $B' \cap (A \cup C)$   
 (d)  $B \cup (A \cup C)'$       (e)  $B \cup (A \cup C)$

$B'$  is everything outside of  $B$ :



we have to add

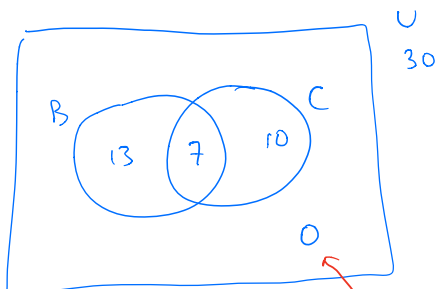


So  $B' \cup (A \cup C)$

Those come from  $A$  and  $C$   
 (the rest of  $A$  and  $C$  are already in  $B'$ )

2. (5 pts.) All 30 students at Gotham University are required to be part of at least one club. They have only two clubs: a book club and a chess club. 20 students join the book club. 7 students join both the book club and the chess club. How many students are in the chess club?  
(Hint: Draw a Venn Diagram.)

- ~~(a)~~ 17      (b) 3      (c) 23      (d) 10      (e) 13



2  
 because every student  
 must be in one of the clubs

Name: \_\_\_\_\_

3. (5 pts.) When ordering a burger at Big Belly Burger, you have to select between 2 different choices of buns. Next, they offer 4 different types of sauces and 3 different choices of cheeses (you can pick any number of sauces and cheeses). Lastly, you can choose to get either a single or double patty. If you want a burger with at least one sauce and at least one type of cheese, how many different burgers are possible?

- (a) 512      ~~(b)~~ 420      (c) 511      (d) 40      (e) 2045

# buns = 2

# choices for sauces  $2^4$  but we have to have at least one so  $2^4 - 1$  (remove  $\emptyset$ )

# choices for cheeses  $2^3 - 1$

# choices for patties 2

$$(2)(2^4 - 1)(2^3 - 1)(2) = (2)(15)(7)(2) = 420$$

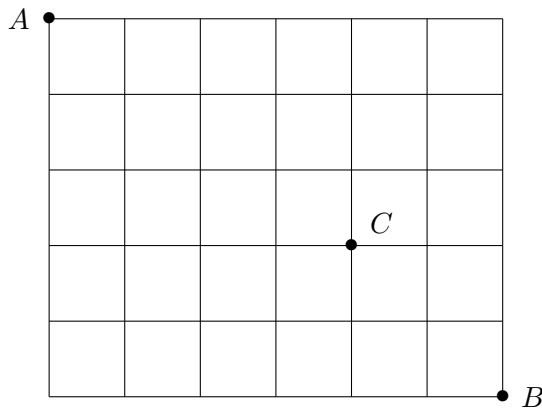
4. (5 pts.) License plates in Star City consist of 3 letters (A - Z) followed by 3 digits (0 - 9), where the letters and digits can be repeated. How many different license plates are possible?

- (a)  $P(26, 3) \cdot P(10, 3)$       (b)  $26^3 + 10^3$       (c)  $C(26, 3) + C(10, 3)$   
(d)  $C(26, 3) \cdot C(10, 3)$       ~~(e)~~  $26^3 \cdot 10^3$

$$\frac{26}{\underline{\quad}} \frac{26}{\underline{\quad}} \frac{26}{\underline{\quad}} - \frac{10}{\underline{\quad}} \frac{10}{\underline{\quad}} \frac{10}{\underline{\quad}}$$

Name: \_\_\_\_\_

5. (5 pts.) The grid below shows a map of Central City. Barry wants to get from  $A$  to  $B$ , but knows that Grodd has set a trap for him at  $C$ . If Barry only moves right or down, how many ways can he get from  $A$  to  $B$  while avoiding  $C$ ?



- (a) 210      (b) 462      (c) 41      ~~(d) 252~~      (e) 421

total # of routes  $C(11, 5) = 462$        $462 - (35)(6) = 252$   
 total # from A to C  $C(7, 3) = 35$   
 total # from C to B  $C(4, 2) = 6$

6. (5 pts.) The 11 members of the MinuteMen wish to get a picture of the group taken. The group consists of 6 men and 5 women. In how many ways can they arrange themselves in a single row if all the men stand together and all the women stand together?

- (a)  $6! + 5!$       (b)  $6! \cdot 5!$       ~~(c)  $2 \cdot 6! \cdot 5!$~~       (d)  $2 \cdot 11!$       (e)  $11!$

You can have women-men or men-women (L to R)

# ways to line women up:  $P(5, 5) = 5!$

" " " " men " :  $P(6, 6) = 6!$

Name: \_\_\_\_\_

7. (5 pts.) A Poker hand consists of 5 cards from a standard deck. A full house consists 3 cards of one rank and 2 cards of another rank, such as three 8s and two 4s; or three aces and two 6s. How many ways are there to get a full house? (Note: Refer to page 19 for a description of the standard deck of cards.)

(a)  $C(13, 3) \cdot C(12, 2)$

(b)  $13 \cdot C(4, 3) + 12 \cdot C(4, 2)$

(c)  $C(13, 2) \cdot C(4, 3) \cdot C(4, 2)$

~~(d)~~  $P(13, 2) \cdot C(4, 3) \cdot C(4, 2)$

(e)  $C(4, 3) \cdot C(4, 2)$

# choices for rank with 3 cards + then rank with 2 cards  $P(13, 2)$

# choices for specific 3 cards  $C(4, 3)$

" " " " 2 "  $C(4, 2)$

8. (5 pts.) If  $S = \{a, b, c\}$  is a sample space, which of the following are valid probability assignments?

I.  $P(a) = \frac{1}{3}, P(b) = \frac{1}{6}, P(c) = \frac{1}{2}$  ok

II.  $P(a) = \frac{1}{3}, P(b) = \frac{2}{3}, P(c) = 0$  ok

III.  $P(a) = 0, P(b) = 1, P(c) = 0$  ok

(a) Only II and III are valid.

(b) Only I is valid.

(c) Only I and III are valid.

~~(d)~~ I, II and III are all valid.

(e) Only I and II are valid.

Name: \_\_\_\_\_

9. (5 pts.) A bag contains 10 balls, of which 7 are red and 3 are blue. Bob randomly chooses two balls from the bag, without replacement. What is the probability that he picks one ball of each color?

- (a)  $\frac{7}{30}$       (b)  $\frac{21}{100}$       ~~(c)  $\frac{7}{15}$~~       (d) 1      (e)  $\frac{8}{15}$

$$\frac{C(7,1) \cdot C(3,1)}{C(10,2)} = \frac{21}{45} = \frac{7}{15}$$

10. (5 pts.) An urn contains seven balls, labelled 1 through 7. Three of these are drawn successively, without replacement, and placed in a row in the order drawn, to form a 3-digit number. (Order is important.) Find the probability that this number is (strictly) less than 320.

- (a)  $\frac{11}{42}$       (b)  $\frac{1}{3}$       ~~(c)  $\frac{13}{42}$~~       (d)  $\frac{2}{7}$       (e)  $\frac{5}{14}$

It could be

1	—	—	# = 6 · 5 = 30	
2	—	—	# = 6 · 5 = 30	
or	3	1	—	# = 5
			<hr/>	
			65	

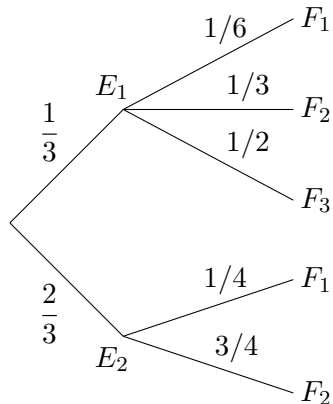
$$prob = \frac{65}{P(7,3)} = \frac{65}{7 \cdot 6 \cdot 5} = \frac{65}{210} = \frac{13}{42}$$

Name: \_\_\_\_\_

11. (5 pts.) Gretchen chooses two cards from a standard deck of 52. (Order is not important and she chooses the cards without replacement.) What is the probability that she chooses one king and one queen? (Note: Refer to page 19 for a description of the standard deck of cards.)

- (a)  $\frac{C(4,1) \cdot C(4,1)}{C(52,2)}$       (b)  $\frac{C(8,2)}{C(52,2)}$       (c)  $\frac{C(4,1)}{C(52,1)} \cdot \frac{C(4,1)}{C(52,1)}$   
 (d)  $\frac{P(4,1) \cdot P(4,1)}{P(52,2)}$       (e)  $\frac{C(52,4) \cdot C(52,4)}{C(52,2)}$

12. (5 pts.) Jeff analyzes a certain carnival game. In the first step the possible outcomes are  $E_1$  and  $E_2$ . In the second step the possible outcomes are  $F_1$ ,  $F_2$  and (possibly)  $F_3$ . Having studied Finite Math, Jeff comes up with the following tree diagram for this game:



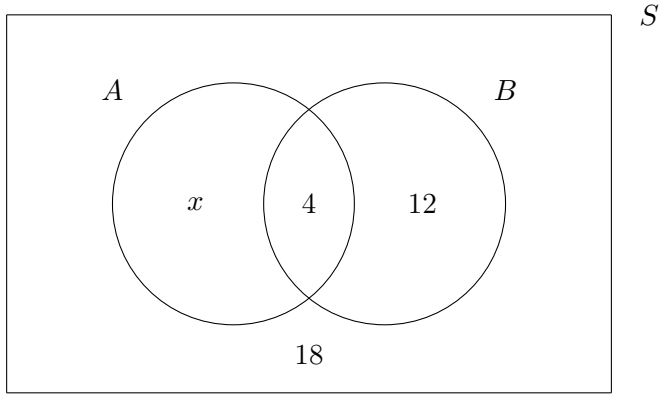
If a player ends up with  $F_2$ , what is the probability that in the first step she got  $E_2$ ?

- (a)  $\frac{2}{3}$       (b)  $\frac{7}{9}$       (c)  $\frac{9}{11}$       (d)  $\frac{11}{13}$       (e)  $\frac{1}{2}$

$$P(E_2 | F_2) = \frac{P(E_2 \cap F_2)}{P(F_2)} = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)}{\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)} = \frac{6/12}{6/12 + 1/9} = \frac{1/2}{1/2 + 1/9} = \frac{1/2}{(9+2)/18} = \frac{1/2}{11/18} = \frac{1}{2} \cdot \frac{18}{11} = \frac{9}{11}$$

Name: \_\_\_\_\_

13. (5 pts.) Consider the following Venn diagram depicting sets  $A$  and  $B$ . (The numbers refer to the number of elements in the sets, not to the elements of the sets.)



What should  $x$  be so that  $A$  and  $B$  are independent?

- (a) 8                      (b) 10                      (c) 4                      (d) 2                      ~~(e) 6~~

$$P(A|B) = \frac{4}{4+12} = \frac{1}{4}$$

$$P(A) = \frac{x+4}{34+x}$$

Set these 2 equal

$$\frac{1}{4} = \frac{x+4}{34+x}$$

$$34+x = 4(x+4)$$

$$= 4x+16$$

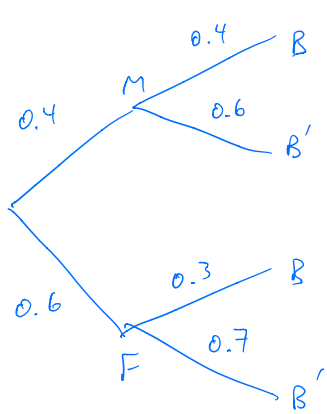
$$34+x = 4x+16$$

$$18 = 3x$$

$$x = 6$$

14. (5 pts.) In a certain group of people, 40% are male and 60% are female. Of the males, 40% have brown eyes. Of the females, 30% have brown eyes. One person is chosen at random and found to have brown eyes. What is the probability that this person is a male? (Hint: Draw a tree diagram.)

- (a)  $\frac{8}{9}$                       (b)  $\frac{2}{5}$                       (c)  $\frac{7}{15}$                       ~~(d)  $\frac{8}{17}$~~                       (e)  $\frac{2}{3}$



$$P(M|B) = \frac{P(M \cap B)}{P(B)} = \frac{(0.4)(0.4)}{(0.4)(0.4) + (0.6)(0.3)}$$

$$= \frac{0.16}{0.16 + 0.18} = \frac{16}{34} = \frac{8}{17}$$



Name: \_\_\_\_\_

15. (5 pts.) The Math 10120 final exam scores for a sample of 10 students are given below. The mean is 120 (you do not have to verify this). Find the sample standard deviation.

95 100 105 115 120 125 125 130 135 150

- (a) 283.33    ~~(b)~~ 16.83    (c) 15.97    (d) 255    (e) 50.50

$$\bar{x} = \frac{95 + \dots + 150}{10} = 120$$

$x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
95	-25	625
100	-20	400
105	-15	225
115	-5	25
120	0	0
125	5	25
125	5	25
130	10	100
135	15	225
150	30	900
		sum = 2550

$$s^2 = \frac{2550}{9} = 283.33$$

↖ not 10

$$s = \sqrt{s^2} = 16.83$$

16. (5 pts.) Consider an experiment where two fair 6 sided dice are rolled. Let  $X$  be the maximum of the two numbers that show up. Find  $P(X > 3)$ .

- ~~(a)~~  $\frac{3}{4}$     (b)  $\frac{1}{2}$     (c)  $\frac{1}{4}$     (d)  $\frac{1}{36}$     (e)  $\frac{8}{9}$

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)  
 (2,1) (2,2) (2,3) (2,4) (2,5) (2,6)  
 (3,1) (3,2) (3,3) (3,4) (3,5) (3,6)  
 (4,1) (4,2) (4,3) (4,4) (4,5) (4,6)  
 (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)  
 (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

the red ones have the max > 3.

$$\text{So } P(X > 3) = \frac{27}{36} = \frac{3}{4}$$

Name: \_\_\_\_\_

17. (5 pts.) 3 balls are picked at random (without replacement) from an urn containing 2 red, 3 blue and 4 green balls. Let  $X$  be the number of red balls picked. Which of the following gives the distribution of  $X$ ?

- (a) 

$x_i$	$P(x_i)$
0	5/12
1	1/2
2	1/12
- (b) 

$x_i$	$P(x_i)$
0	5/12
1	1/2
2	1/24
3	1/24
- (c) 

$x_i$	$P(x_i)$
0	2/9
1	3/9
2	4/9
- (d) 

$x_i$	$P(x_i)$
0	1/4
1	1/4
2	1/4
3	1/4
- (e) 

$x_i$	$P(x_i)$
0	1/3
1	1/3
2	1/3

$P(X=0) = \frac{C(7,3)}{C(9,3)} = \frac{35}{84} = \frac{5}{12}$  ← (At this point you know it's (a) because the possible values of  $X$  are 0, 1, 2.)

$P(X=1) = \frac{C(2,1) \cdot C(7,2)}{C(9,3)} = \frac{2 \cdot 21}{84} = \frac{1}{2}$  ,  $P(X=2) = \frac{C(2,2) \cdot C(7,1)}{C(9,3)} = \frac{7}{84} = \frac{1}{12}$

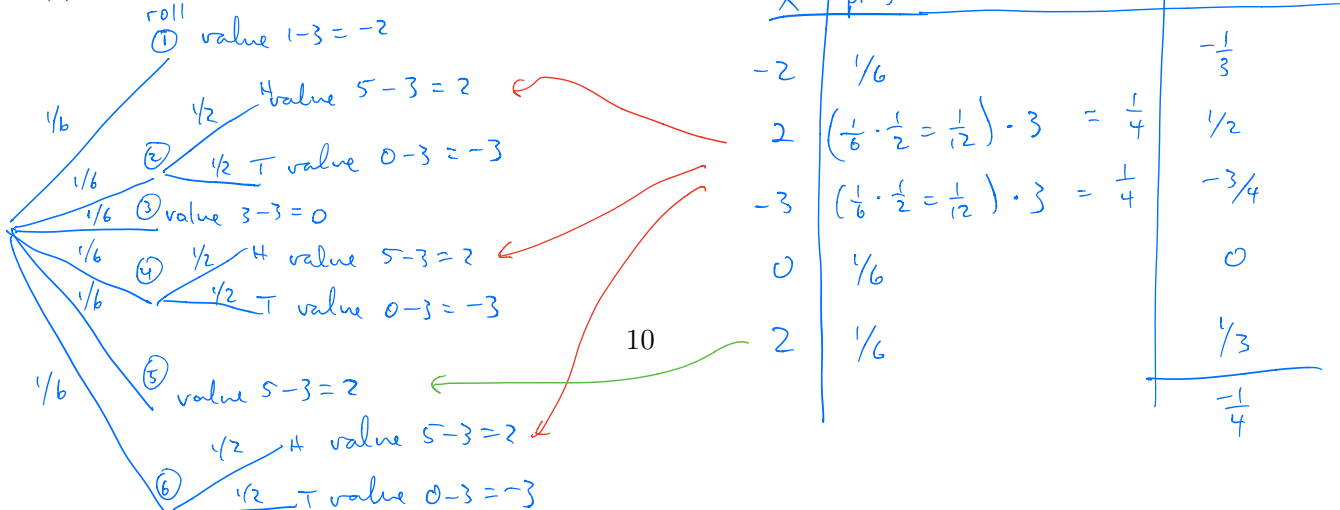
18. (5 pts.) Alice plays the following carnival game that costs \$3. She starts by tossing a fair six-sided die.

- If the number that shows up is odd, then Alice gets that amount of money.
- If the number is even, then Alice tosses a coin.
  - If the coin toss results in a heads, then Alice gets \$5.
  - If the coin toss results in tails, then Alice gets nothing.

How much money does Alice expect to earn? (Remember to take into account that she pays \$3 to play.)

- (a) \$2.75      ~~(b) -\$0.25~~      (c) \$1.0      (d) -\$0.75

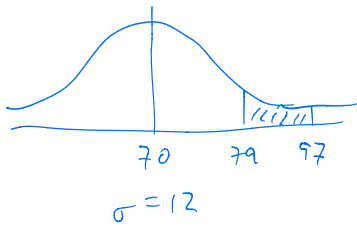
(e) The game is fair.



Name: \_\_\_\_\_

19. (5 pts.) Every time Alice takes a bite of the "Eat Me" cake, she grows taller by a random number of centimeters, which we will refer to as  $X$ . Suppose that  $X$  can be modelled by the Normal distribution with  $\mu = 70$  and  $\sigma = 12$ . Find  $P(79 \leq X \leq 97)$ .

- (a) 0.7734      (b) 0.2266      ~~(c)~~ 0.2144      (d) 0.7612      (e) 0.4878



$$z_1 = \frac{79 - 70}{12} = \frac{9}{12} = 0.75$$

$$z_2 = \frac{97 - 70}{12} = \frac{27}{12} = \frac{9}{4} = 2.25$$

$$A(2.25) - A(0.75) = 0.2144$$

20. (5 pts.) Green Arrow has 20 arrows and he hits a target with probability 0.8. Suppose he shoots all his arrows and let  $X$  be the number of arrows that hit the target. What is the probability that  $X$  is at least 10 and at most 12?

- (a)  $(0.8)^{11}(0.2)^9$   
 (b)  $C(20, 10)(0.8)^{10}(0.2)^{10} + C(20, 12)(0.8)^{12}(0.2)^8$   
~~(c)~~  $C(20, 10)(0.8)^{10}(0.2)^{10} + C(20, 11)(0.8)^{11}(0.2)^9 + C(20, 12)(0.8)^{12}(0.2)^8$   
 (d)  $C(20, 11)(0.8)^{11}(0.2)^9$   
 (e)  $(0.8)^{10}(0.2)^{10} + (0.8)^{11}(0.2)^9 + (0.8)^{12}(0.2)^8$

$$P(X=10) = C(20, 10) (0.8)^{10} (0.2)^{10}$$

$$P(X=11) = C(20, 11) (0.8)^{11} (0.2)^9$$

$$P(X=12) = C(20, 12) (0.8)^{12} (0.2)^8$$

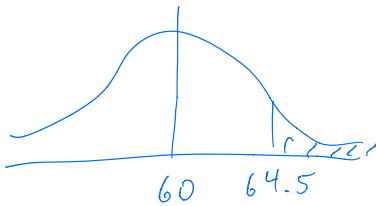
Name: \_\_\_\_\_

21. (5 pts.) Suppose now that Green Arrow goes to battle with 100 arrows and that the probability that an arrow he shoots hits its target is 0.6. Let  $X$  be the number of arrows that hit their targets. Using the Normal distribution (i.e., the Normal approximation to the Binomial distribution), estimate  $P(X \geq 65)$ . (Note: Don't worry too much about rounding errors and pick the closest answer.)

- (a) 82%      (b) 42%      ~~(c) 18%~~      (d) 58%      (e) 32%

$$\mu = np = 100(0.6) = 60 \qquad n=100 \quad p=0.6 \quad q=0.4$$

$$\sigma = \sqrt{npq} = \sqrt{100(0.6)(0.4)} = \sqrt{24} = 4.90$$



$$z = \frac{64.5 - 60}{4.90} = \frac{4.5}{4.90} = 0.92$$

$$0.5 - A(0.92) = 0.1788$$

↑  
notice we bump it down by 0.5

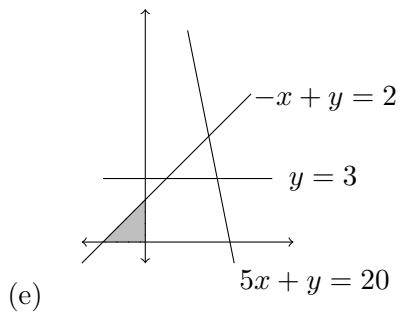
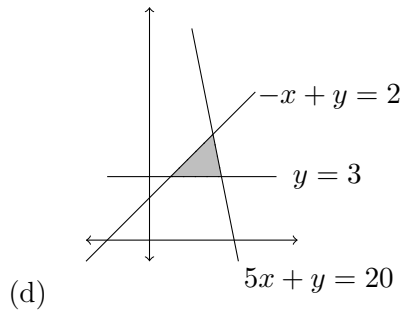
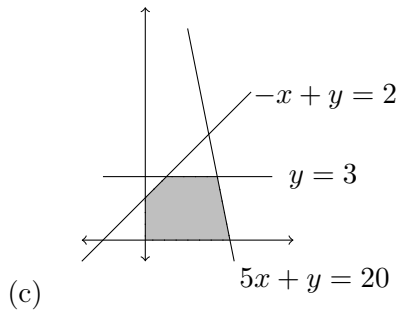
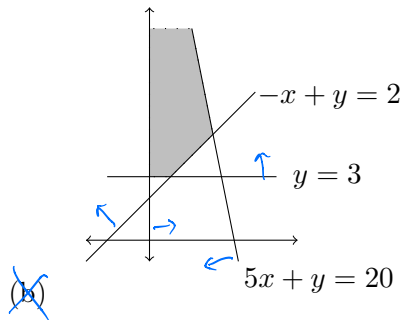
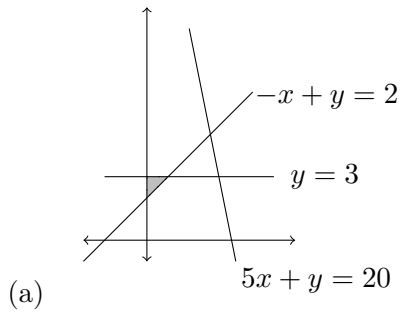
Name: \_\_\_\_\_

22. (5 pts.) Consider the system of inequalities

$$\begin{aligned} -x + y &\geq 2 \\ 5x + y &\leq 20 \\ x &\geq 0 \\ y &\geq 3 \end{aligned}$$

*test point (0,0) fails  
(0,3) works*

Which of the following shaded regions is the feasible set for this system of linear inequalities?



Name: \_\_\_\_\_

**23.** (5 pts.) Tricia's Trikes produces two kinds of tricycles, regular and deluxe. With the upcoming holiday season near at hand, they want to produce at least 200 tricycles a day. For budgetary reasons they can spend at most \$50,000 per day on production, and because of limitations on their trucks the total weight of the tricycles each day has to be at most 4,500 pounds. Each regular tricycle costs \$30 to produce and weighs 10 pounds. Each deluxe tricycle costs \$40 to produce and weighs 15 pounds. Find the inequalities relating the number of regular tricycles produced ( $x$ ) and the number of deluxe tricycles produced ( $y$ ). (Warning: pay close attention to the direction of the inequalities when you choose your answer, and pay attention to "at most" versus "at least" in the statement of the problem.)

(a) 
$$\begin{aligned} x + y &\leq 200 \\ 30x + 40y &\geq 50,000 \\ 10x + 15y &\leq 4,500 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

~~(b)~~

$$\begin{aligned} x + y &\geq 200 \\ 30x + 40y &\leq 50,000 \\ 10x + 15y &\leq 4,500 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(c) 
$$\begin{aligned} x + y &\leq 200 \\ 30x + 40y &\leq 50,000 \\ 10x + 15y &\leq 4,500 \end{aligned}$$

(d)

$$\begin{aligned} x + y &\leq 200 \\ 30x + 40y &\leq 50,000 \\ 10x + 15y &\leq 4,500 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(e) 
$$\begin{aligned} x + y &\geq 200 \\ 30x + 40y &\leq 50,000 \\ 10x + 15y &\geq 4,500 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

produce at least 200 tricycles —  $x + y \geq 200$   
 spend at most 50,000 —  $30x + 40y \leq 50,000$   
 weight at most 4500 —  $10x + 15y \leq 4500$   
 $x$  and  $y$  can't be negative —  $x \geq 0, y \geq 0$

Name: \_\_\_\_\_

*maximum*

24. (5 pts.) Find the ~~maximize~~ value of  $z = 3x + 4y$  subject to the constraints

$$6x + 3y \leq 18$$

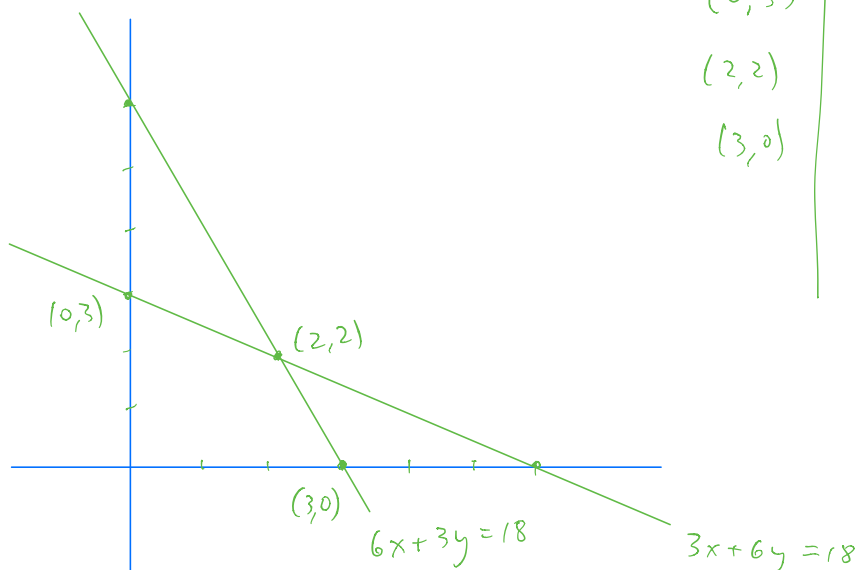
$$3x + 6y \leq 18$$

$$x \geq 0$$

$$y \geq 0$$

- (a) 0      ~~(b)~~ 14      (c) 9      (d) 12  
 (e) There is no maximum.

point	$z = 3x + 4y$
$(0, 3)$	12
$(2, 2)$	14
$(3, 0)$	9



$$\left. \begin{array}{l} 6x + 3y = 18 \\ 3x + 6y = 18 \end{array} \right\} \rightarrow \begin{array}{l} 6x + 3y = 18 \\ 6x + 12y = 36 \\ \hline 9y = 18 \\ y = 2 \\ x = 2 \end{array}$$

Name: \_\_\_\_\_

25. (5 pts.) Evaluate the following expression:

$$\left( \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

- (a)  $\begin{bmatrix} 6 \\ 10 \end{bmatrix}$       (b)  $\begin{bmatrix} 18 \\ -12 \end{bmatrix}$       ~~(c)~~  $\begin{bmatrix} 8 \\ 7 \end{bmatrix}$       (d)  $\begin{bmatrix} 17 \\ -5 \end{bmatrix}$       (e)  $\begin{bmatrix} 18 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 12-6 \\ 4+6 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

26. (5 pts.) Roosevelt and Churchill play a game using the following payoff matrix:

$$\begin{bmatrix} 4 & 6 & 5 \\ 3 & 8 & 2 \\ 3 & 2 & 6 \end{bmatrix}$$

Find the value of this game.

- (a) 3      ~~(b)~~ 4      (c) 6      (d) 2  
(e) This game is not strictly determined.

*R's calculation*

$$\begin{bmatrix} \boxed{4} & 6 & 5 \\ 3 & 8 & \textcircled{2} \\ 3 & \textcircled{2} & 6 \end{bmatrix}$$

*C's calculation*

$$\begin{bmatrix} \boxed{4} & 6 & 5 \\ 3 & \textcircled{8} & 2 \\ 3 & 2 & \textcircled{6} \end{bmatrix}$$



Name: \_\_\_\_\_

27. (5 pts.) Russia and China play a game with payoff matrix:

$$\begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix}$$

If Russia's strategy is  $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$  and China's strategy is  $\begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$ , what is the expected payoff of this game.

- (a)  $-\frac{1}{2}$       (b) 1      (c) 0      (d) -1      (e)  $\frac{1}{2}$

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1+0+0 \\ -\frac{3}{2}+0+\frac{3}{2} \\ 0+0-1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

28. (5 pts.) Suppose we have a game with payoff matrix:

$$\begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} C \\ c_1 \quad c_2 \quad c_3 \\ \\ \\ \end{array} \\ \begin{array}{c} R \\ r_1 \\ r_2 \\ r_3 \end{array} \begin{bmatrix} 3 & -2 & 1 \\ 4 & 2 & -6 \\ -2 & 3 & -4 \end{bmatrix}$$

If  $R$  uses the pure strategy of always choosing  $r_2$ , and  $C$  uses the mixed strategy of choosing  $c_1$  50% of the time and  $c_3$  50% of the time, what is the expected payoff of this game?

- (a) 5      (b) -2      (c) -1      (d) 0      (e) 4

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 4 & 2 & -6 \\ -2 & 3 & -4 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \end{bmatrix}$$

Name: \_\_\_\_\_

29. (5 pts.) The game with payoff matrix:

$$\begin{bmatrix} 3 & -6 \\ 2 & 4 \end{bmatrix}$$

is not strictly determined. If both players use the optimal strategy, what is the value (expected payoff) of this game? (See formulas on page 19.)

- (a) -24      (b) 0      (c)  $\frac{16}{11}$       (d)  $\frac{24}{11}$       (e)  $\frac{3}{11}$

$$v = \frac{12 - (-12)}{3+4 - (-6) - 2} = \frac{24}{11}$$

30. (5 pts.) Find Robin's optimal strategy for the following game:

$$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & -2 \\ -1 & 1 & -3 \end{bmatrix}$$

(Note: Formulas for computing optimal mixed strategies for  $2 \times 2$  matrices are given on page 19.)

- (a)  $[1/3 \ 1/3 \ 1/3]$       (b)  $[1/2 \ 0 \ 1/2]$   
 (c)  $[2/3 \ 1/3 \ 0]$       (d)  $[2/3 \ 0 \ 1/3]$   
 (e)  $[1/2 \ 1/2 \ 0]$

R would never choose  $r_3$  because  $r_2$  dominates it ( $2 > -1$ ,  $3 > 1$ ,  $-2 > -3$ )  
 So remove  $r_3$ :  $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & -2 \end{bmatrix}$ . Now C would never choose  $c_2$  because  $c_1$  dominates it ( $-1 < 0$ ,  $2 < 3$ ). So remove  $c_2$ :  $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$ . See if this strictly determined:

R's calc:  $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$   
 C's calc:  $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$

Now use formula (next page) for R:

$$p_1 = \frac{-2 - 2}{-1 + (-2)} = \frac{-4}{-3} = \frac{4}{3}$$

18

$\begin{bmatrix} 1/2 & 1/2 & 0 \end{bmatrix}$   
 since R would never choose  $r_3$

so not strictly determined.

Name: \_\_\_\_\_

### Game Theory Formulas.

For a mixed strategy game with payoff matrix

$$\begin{array}{c} R \\ r_1 \\ r_2 \end{array} \begin{array}{cc} C \\ c_1 & c_2 \\ \left[ \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right] \end{array}$$

R's optimal strategy is given by  $[p_1 \ p_2]$  where

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \text{ and } p_2 = 1 - p_1.$$

C's optimal strategy is given by  $\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ , where

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \text{ and } q_2 = 1 - q_1.$$

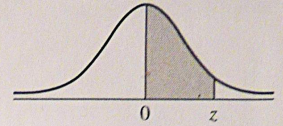
If both players play their optimal strategies, then the expected payoff (or value of the game) is

$$v = \frac{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}.$$

### Standard Deck.

Each card has a rank and a suit. There are 13 possible ranks (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K) and 4 possible suits (Clubs, Diamonds, Hearts and Spades). There are 4 cards of each rank, 13 cards of each suit and  $13 \cdot 4 = 52$  cards in total.

## Area Under the Standard Normal Curve



$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		



**Math 10120 Finite Math.**  
**Practice Final Exam 2**  
**May 8, 2019**

Name: \_\_\_\_\_

- Be sure that you have all 20 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (a) (●) (c) (d) (e)  | 17. (●) (b) (c) (d) (e) |
| 2. (●) (b) (c) (d) (e)  | 18. (a) (●) (c) (d) (e) |
| .....                   |                         |
| 3. (a) (●) (c) (d) (e)  | 19. (a) (b) (●) (d) (e) |
| 4. (a) (b) (c) (d) (●)  | 20. (a) (b) (●) (d) (e) |
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| 5. (a) (b) (c) (●) (e)  | 21. (a) (b) (●) (d) (e) |
| 6. (a) (b) (●) (d) (e)  | 22. (a) (●) (c) (d) (e) |
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| 7. (a) (b) (c) (●) (e)  | 23. (a) (●) (c) (d) (e) |
| 8. (a) (b) (c) (●) (e)  | 24. (a) (●) (c) (d) (e) |
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| 9. (a) (b) (●) (d) (e)  | 25. (a) (b) (●) (d) (e) |
| 10. (a) (b) (●) (d) (e) | 26. (a) (●) (c) (d) (e) |
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| 11. (●) (b) (c) (d) (e) | 27. (a) (b) (●) (d) (e) |
| 12. (a) (b) (●) (d) (e) | 28. (a) (b) (●) (d) (e) |
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| 13. (a) (b) (c) (d) (●) | 29. (a) (b) (c) (●) (e) |
| 14. (a) (b) (c) (●) (e) | 30. (a) (b) (●) (d) (e) |
| .....                   |                         |
| 15. (a) (●) (c) (d) (e) |                         |
| 16. (●) (b) (c) (d) (e) |                         |