Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2020

Name:________________________________

Instructor: Juan Migliore

Exam 2 Solutions

March 5, 2020

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You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an × through your answer to each problem.

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9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
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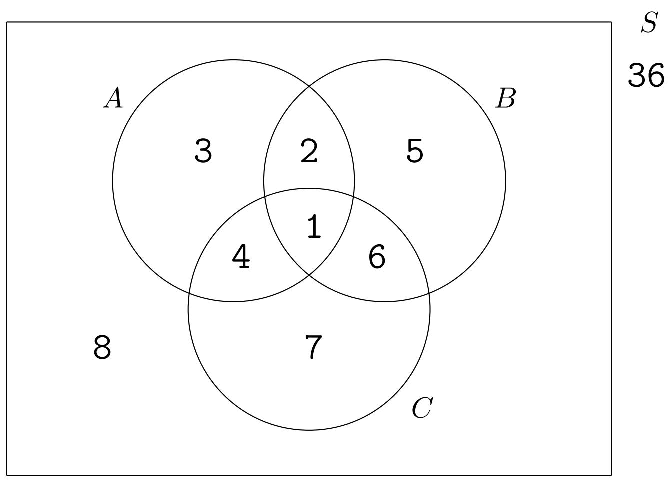
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B) and Colombia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability that she is going to exactly two of the countries (not one, not three)?



(a) $\frac{15}{36}$ (b) $\frac{13}{36}$ ~~(c)~~ $\frac{12}{36}$ (d) $\frac{27}{36}$ (e) $\frac{1}{36}$

$\frac{2+4+6}{36} = \frac{12}{36} = \frac{1}{3}$

2. (5 pts.) Referring to the Venn diagram in problem #1, find $P(A'|B)$.

(a) $\frac{3}{14}$ (b) $\frac{11}{36}$ (c) $\frac{11}{26}$ (d) $\frac{5}{14}$ ~~(e)~~ $\frac{11}{14}$

$P(A'|B) = \frac{11}{14}$

since B contains $5+2+1+6=14$ people, of whom $5+6$ are not in A.

3. (5 pts.) Suppose A, B, C are three events with the following probability information:

$$P(A) = \tfrac{1}{2}, \qquad P(B) = \tfrac{1}{3}, \qquad P(C) = \tfrac{1}{4}$$

$$P(A \cap B) = \tfrac{5}{6}, \qquad P(A \cap C) = \tfrac{1}{2}, \qquad P(B \cap C) = \tfrac{1}{12}$$

Which pairs of events are independent?

~~(a)~~ B and C are independent $P(B) \cdot P(C) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12} = P(B \cap C)$ indep

(b) A and B are independent $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \neq P(A \cap B)$ not indep

(c) A and C are independent $P(A) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \neq P(A \cap C)$ not indep

(d) A and C are independent, and also B and C are independent

(e) A and B are independent, and also A and C are independent

4. (5 pts.) The Dakota Club has six members from North Dakota and two members from South Dakota. Two members are chosen at random to represent the club at a national conference. What is the probability that the two chosen members are from the same state?

~~(a)~~ $\frac{4}{7}$ (b) $\frac{15}{28}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$ (e) $\frac{3}{4}$

$$\frac{\binom{6}{2} + \binom{2}{2}}{\binom{8}{2}} = \frac{15+1}{28} = \frac{16}{28} = \frac{4}{7}$$

5. (5 pts.) In a certain town in Canada, 50% of the population speaks English, 70% speaks French and 30% speaks both English and French. A person is chosen at random, and it is noted that she speaks English. What is the probability that she also speaks French?

(a) 70% ~~(b)~~ 60% (c) 30% (d) 40% (e) 20%

$$P(F \mid E) = \frac{P(F \cap E)}{P(E)} = \frac{.30}{.50} = \frac{3}{5} = 60\%$$

6. (5 pts.) Claire belongs to a certain club that has a total of 7 members. The club is supposed to choose a team of three people to represent the club in a national conference. If the three are chosen at random, what is the probability that Claire is one of the three chosen?

(a) $\frac{1}{7}$ (b) $\frac{1}{3}$ (c) $\frac{3}{35}$ (d) $\frac{1}{35}$ ~~(e)~~ $\frac{3}{7}$

$C(6,2)$ ← choose Claire plus two more

$C(7,3)$ ← all ways of choosing 3

$$= \frac{15}{35} = \frac{3}{7}$$

7. (5 pts.) On a certain large plane, 8 passengers are sitting in row 52. Of these, 6 chose broccoli for their vegetable at supper and 2 chose cauliflower. Of the broccoli passengers, 4 watched a movie after supper and 2 took a nap. Of the cauliflower passengers, both took naps after supper. The flight crew randomly chose a passenger from row 52 to ask about the flight, and learned that that passenger took a nap after supper. What is the probability that that passenger also had broccoli?

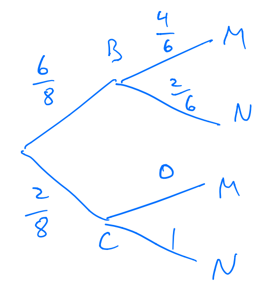
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ ~~(d)~~ $\frac{1}{2}$ (e) $\frac{1}{4}$

B = chose broccoli

C = chose cauliflower

M = movie

N = nap



$$P(B|N) = \frac{P(B\cap N)}{P(N)}$$

$$= \frac{\frac{6}{8}\cdot\frac{2}{6}}{\frac{6}{8}\cdot\frac{2}{6} + \frac{2}{8}\cdot 1}$$

$$= \frac{1/4}{\frac{1}{4}+\frac{1}{4}} = \frac{1}{2}$$

8. (5 pts.) Emily is given a fair coin and told that she can flip it up to four times. She stops when it lands heads, or after the fourth flip. She wins if it ever lands heads. Otherwise she loses. What is the probability that she wins?

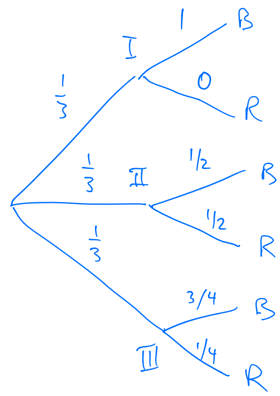
(a) $\frac{7}{8}$ (b) $\frac{3}{4}$ ~~(c)~~ $\frac{15}{16}$ (d) $\frac{1}{2}$ (e) $\frac{1}{16}$

$$1 - P(\text{loses}) = 1 - \left(\tfrac{1}{2}\right)^4 = \frac{15}{16}$$

You could also make a tree diagram and solve it directly.

9. (5 pts.) I have three boxes. Box I has 3 blue balls and 0 red ones. Box II has 2 blue balls and 2 red ones. Box III has 6 blue balls and 2 red ones. I choose a box at random and draw a ball from it. If the ball I draw is blue, what is the probability that it came from box III?

~~(a)~~ $\frac{1}{3}$ (b) $\frac{6}{11}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{2}{3}$



$$P(III \mid B) = \frac{P(III \cap B)}{P(B)}$$

$$= \frac{(\frac{1}{3})(\frac{3}{4})}{\frac{1}{3}(1) + \frac{1}{3}(\frac{1}{2}) + \frac{1}{3}(\frac{3}{4})}$$

$$= \frac{1/4}{\frac{1}{3} + \frac{1}{6} + \frac{1}{4}} = \frac{3/12}{9/12} = \frac{3}{9} = \frac{1}{3}$$

10. (5 pts.) You are dealt 5 cards from a standard deck of 52 cards (without replacement). What is the probability that you get a full house (i.e. what is the probability that you get three of one rank and two of another; e.g. three kings and two 3's)? Remember that there are 13 ranks: A,2,3,4,5,6,7,8,9,10,J,Q,K.

(a) $\frac{13 \cdot C(4,3) \cdot C(4,2)}{C(52,5)}$ (b) $\frac{C(13,2)}{C(52,5)}$ ~~(c)~~ $\frac{13 \cdot C(4,3) \cdot 12 \cdot C(4,2)}{C(52,5)}$

(d) $\frac{C(4,3) \cdot C(4,2)}{C(52,5)}$ (e) $\frac{13 \cdot 12 \cdot C(52,3) \cdot C(52,2)}{C(52,5)}$

which rank do you have three of?

which rank do you have two of?

choose three of the four

choose two of the four

$$\frac{13 \cdot 12 \cdot C(4,3) \cdot C(4,2)}{C(52,5)}$$

choose 5 cards from the deck

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. **You're more likely to get partial credit for a wrong answer if you explain your reasoning**.

11. (10 pts.) **Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k). Be sure to mark your answer.

Amy, Bernie, Elizabeth, Joe, Mike, Pete and Tom are running in a race. Assume that they are equally likely to win, and that there can be no ties.

(a) What is the probability that the girls (Amy and Elizabeth) both finish before all the boys. [Remember to take into account whether Amy is first and Elizabeth second, or vice versa.]

the order that Amy and Elizabeth finish in 1st two

the order that the five guys finish

$$\frac{2 \cdot 5!}{7!}$$

all possible orders for the seven runners

Answer to (a):

(b) What is the probability that the seven runners finish in alphabetical order (i.e. in the order stated above)?

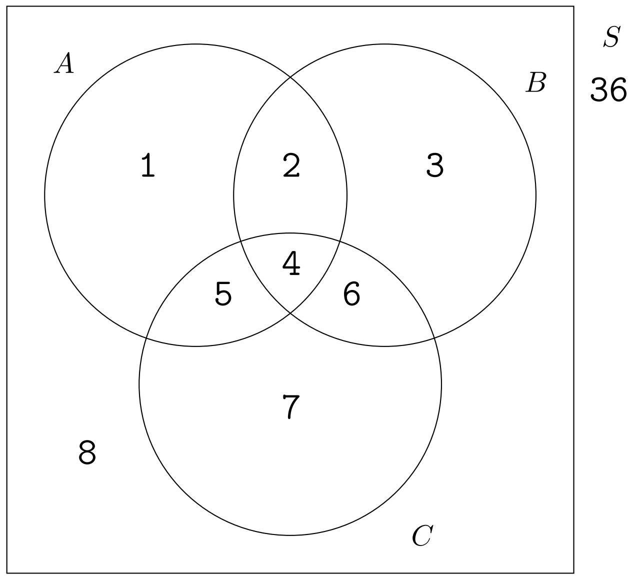
only one order is alphabetical

$$\frac{1}{7!}$$

all possible orders

Answer to (b):

12. (10 pts.) The following Venn diagram describes the relative sizes of events A, B and C in a sample space S.



Find each of the probabilities using the numbers in the diagram. For example, if it asked for $P(A)$ you could write any one of the following:

$$\frac{1}{3} \quad \text{or} \quad \frac{12}{36} \quad \text{or} \quad \frac{1+2+4+5}{36}.$$

(a) $P(A \cup B) =$

$\frac{1+2+3+4+5+6}{36} = \frac{21}{36} = \frac{7}{12}$

(b) $P(B \mid A) =$ $\frac{2+4}{1+2+4+5} = \frac{6}{12} = \frac{1}{2}$

(c) $P(A \cap B' \cap C) =$ $\frac{5}{36}$

"in A and C but not in B"

(d) $P(A \cap B \mid C) =$ $\frac{4}{4+5+6+7}$

$= \frac{4}{22} = \frac{2}{11}$

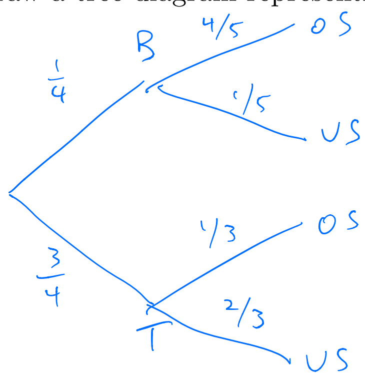
(e) $P(A \cap B \mid C') =$ $\frac{2}{1+2+3+8}$

$= \frac{2}{14} = \frac{1}{7}$

13. (10 pts.) One quarter (1/4) of the people in a certain city prefer to read physical books (B) while three quarters (3/4) prefer to read some sort of tablet (T).

- Of the people who prefer physical books, four fifths (4/5) are over 60 years old (OS) and one fifth (1/5) are under 60 (US).
- Of the people who prefer tablets, one third (1/3) are over 60 (OS) and two thirds (2/3) are under 60 (US).

(a) Draw a tree diagram representing this situation.



(b) If a person in that city is chosen at random, find the probability that he/she is over 60.

$$P(OS) = \left(\frac{1}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)$$

$$= \frac{1}{5} + \frac{1}{4} = \frac{9}{20} = 0.45$$

(c) A person in that city is chosen at random, and it is determined that he/she is over 60. Find the probability that he prefers to read physical books. Show your work!!!

$$P(B \mid OS) = \frac{P(B \cap OS)}{P(OS)} = \frac{\left(\frac{1}{4}\right)\left(\frac{4}{5}\right)}{9/20} = \frac{4}{9}$$

from (b)

14. (10 pts.) There are 6 students in a certain seminar, and each is asked for his/her birth **month**. Assume for simplicity that all months are equally likely, so the only information you need is that there are 12 months.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k). Be sure to mark your answer.

(a) (6 pts.) What is the probability that all 6 of the students were born in **different** months?

12 choices for 1st person, 11 for 2nd person, 10 for 3rd etc so they're all different

$$\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12}$$

12 choices for each person

$$= \frac{P(12,6)}{12^6}$$

Answer to (a):

(b) (4 pts.) What is the probability that **at least** two of the students were born in the same month? [Hint: use your answer to (a).]

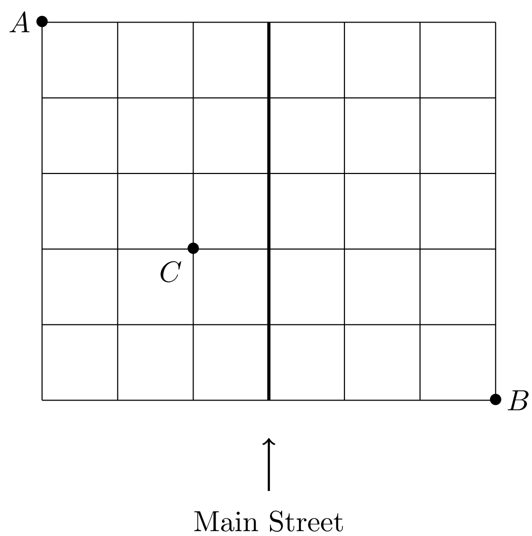
Complement – if they're not all different then at least two are the same

Answer to (b):

$$1 - \frac{P(12,6)}{12^6}$$

15. (10 pts.) **Note:** In this problem, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k). Be sure to show all your work and be sure to plainly mark your answer.

The following is a street map of part of a city. Aaron starts at corner A, and is planning to go all the way to Bob's house at B. At each intersection he only goes east (right) or south (down). He makes that choice randomly unless there is only one choice. (E.g. at the top rightmost corner he only has one choice, namely south.) Main Street is the street marked in boldface.



(a) How many routes are there to get to Bob's house? (Main Street plays no role in this part.)

There are 11 blocks, of which you have to choose 5 to go south

so $C(11, 5)$

(b) Claire lives at the intersection marked C. What is the probability that Aaron's route passes by Claire's house? (Main Street plays no role in this problem either.)

$$\frac{C(5,2) \cdot C(6,2)}{C(11,5)}$$

$C(5,2)$ routes from A to C

$C(6,2)$ routes from C to B

(c) What is the probability that Aaron's route includes all five blocks of Main Street? [Hint: how many routes include all five blocks of Main Street?]

To go the full length of Main Street you have to go the whole way from top to bottom. There's only one route that does that, namely EEESSSSSEEE. So answer is:

$$\frac{1}{C(11,5)}$$

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